Delft University of Technology, EEMCS faculty
Examination Mathematics 2, AESB1210 (test 2)
Friday, January 9th, 2015, 13.45-15.45

- It's not allowed to use a calculator or a mathematical table.
- Each answer should be clearly motivated.
- Your grade is obtained by rounding (score+4)/4 to one decimal place.
- Points:

Ex. 1	4	Ex. 2	4	Ex. 3	3	Ex. 4	4	Ex. 5a	2
								Ex. 5b	2
								Ex. 5c	2
Ex. 6	4	Ex. 7	4	Ex. 8	3	Ex. 9	4		

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1. Suppose a, b and c are real constants such that a is not zero and the system

$$\begin{cases} x_1 + x_2 + x_3 & = f \\ x_1 + (a+1)x_2 + 3x_3 & = g \\ bx_2 + cx_3 & = h \end{cases}$$

is consistent for all possible values of f, g and h. What does this imply for the numbers a, b and c?

2. Find the solutions of the linear system whose augmented matrix is given by

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

and write this solution set in *parametric vector form.*

3. Let $\underline{a}_1$, $\underline{a}_2$ and $\underline{a}_3$ be vectors in $\mathbb{R}^n$ such that $5\underline{a}_2 = 2\underline{a}_1 - 4\underline{a}_3$ and $A = \begin{bmatrix} \underline{a}_1 & \underline{a}_2 & \underline{a}_3 \end{bmatrix}$, so $\underline{a}_1$, $\underline{a}_2$ and $\underline{a}_3$ are the columns of matrix A. Find a solution of the homogeneous linear system $A\underline{x} = \underline{0}$. (Hint: recall the definition of the product of a matrix and a vector)

4. Determine the value(s) of a such that $\left\{ \begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} a \\ a+2 \end{bmatrix} \right\}$ is linearly independent.

p.t.o.

5. Let $A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$ and let T be the corresponding matrix transformation.

- **a.** Find all vectors $\underline{x}$ that are mapped into $\underline{0}$ by transformation T (so find the null space of T).
- **b.** Is transformation T onto?
- **c.** Is transformation T one-to-one?

6. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation that first reflects vectors about the x_1x_2 – plane and then rotates vectors about the x_2 – axis through π radians.
Find the standard matrix of T.

7. Determine a and d such that matrix $A = \begin{bmatrix} a & 0 \\ 1 & d \end{bmatrix}$ has the property $A^2 = A$.

8. Find $x \in \mathbb{R}$ such that $\begin{bmatrix} 2x & 7 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$.

9. Prove or disprove: if A is an 2×2 – matrix such that $A^2 = 0$ then $A = 0$ (of course by 0 is meant the zero marix with size 2×2).