Delft University of Technology, EEMCS faculty
Examination Mathematics 2, AESB1210 (test 2)
Friday, April 17th, 2015, 9.00-12.00

- It's not allowed to use a calculator or a mathematical table.
- Each answer should be clearly motivated.
- Note: Each test takes 1 hour so should be submitted after 60 minutes. After handing in your working out of a test it's allowed to take another test.
- Your grade is obtained by rounding (score+3)/3 to one decimal place.
- Points:

Ex. 1	3	Ex. 2	4	Ex. 3	3	Ex. 4	3	Ex. 5a+5b	6	Ex. 6	3	Ex. 7	3	Ex. 8	2

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1. Suppose a and b are real constants and the system

$$\begin{cases} 2x_1 + 4x_2 + x_3 & = f \\ ax_1 + 9x_2 + 3x_3 & = g \\ 6x_1 + 12x_2 + bx_3 & = h \end{cases}$$

is consistent for all possible values of f, g and h. What does this imply for the coeficients a and b ?

2. Let $\underline{a}_1, \underline{a}_2$ and $\underline{a}_3$ be vectors in $\mathbb{R}^n$ such that $\{\underline{a}_1,\ \underline{a}_2\}$ is linearly independent and $\underline{a}_3 = 5\underline{a}_1 - 8\underline{a}_2$. Suppose $A = [\underline{a}_1\ \underline{a}_2\ \underline{a}_3]$, so $\underline{a}_1$, $\underline{a}_2$ and $\underline{a}_3$ are the columns of matrix A, and $\underline{b} = \underline{a}_1 - \underline{a}_2 + 6\underline{a}_3$.
Write the solution set of $A\underline{x} = \underline{b}$ in *parametric vector form*.

3. *Prove or disprove*: If $\underline{v}_1$ and $\underline{v}_2$ are two vectors in $\mathbb{R}^n$ and $H = Span\,\{\underline{v}_1 + \underline{v}_2, \underline{v}_1 - \underline{v}_2\}$ then $\underline{v}_2 \in H$.

4. If the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, find $T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right)$.

5. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be given by $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} a+b+2c+3d \\ 2c+2d \\ a+b+c+2d \end{bmatrix}$.

a. Investigate whether transformation T is onto.

b. Investigate whether transformation T is one-to-one.

p.t.o.

6. Solve the matrix equation $A(B + CX) = D$ for X (You may assume that all the matrices are invertible $n \times n$ – matrices).

7. *Prove or disprove:* If matrix A is invertible and if (real number) $r \neq 0$ then matrix rA is invertible and $(rA)^{-1} = rA^{-1}$.

8. Suppose that M is an invertible matrix such that the inverse of $5M$ is $\begin{bmatrix} 5 & 6 \\ 5 & 5 \end{bmatrix}$.

Find M.