

- It's not allowed to use a calculator or a mathematical table.
- Each answer should be clearly motivated.
- Note: Each test takes 1 hour so should be submitted after 60 minutes. After handing in your working out of a test it's allowed to take another test.
- Your grade is obtained by rounding $(\text{score} + 2\frac{1}{2}) / 2\frac{1}{2}$ to one decimal place.
- Points:

Ex. 1	$3\frac{1}{2}$	Ex. 2	6	Ex. 3	4	Ex. 4	5	Ex. 5	4
-------	----------------	-------	---	-------	---	-------	---	-------	---

1. Let $y(t)$ be the solution of the initial value problem $y' = 3 - 5t - \frac{1}{2}y$ and $y(0) = 1$. Use Euler's method with $h = 0.2$ to approximate $y(0.4)$.
2. Express the complex number $\frac{(1-i)^8}{(1+i)^6}$ in the form $x + yi$ where $x, y \in \mathbb{R}$.
3. Find $|e^z|$ if $z = e^{\frac{\pi}{4}i}$.
4. Newton's cooling law states that if a hot object is placed into a cool environment the object will cool at a rate proportional to the difference in temperature between the object and the environment. In a similar way a cold object, placed into a warm environment, will warm. This leads to the following mathematical model
$$\frac{dT(t)}{dt} = -k(T(t) - T_e)$$
where $T(t)$ is the temperature of the object at time t , T_e is the temperature of the environment (which is assumed to be constant) and k is a positive constant of proportionality (so $-k$ is negative). Now we suppose that the temperature of a cup of coffee obeys Newton's law of cooling. Take $k = 0.1$, $T_e = 70^\circ\text{F}$ and determine $T(t)$, the temperature of the coffee after t minutes, if the coffee has a temperature of 200°F when freshly poured.
5. Find a particular solution of $y'' + 4y = t^2 + 3e^t$.