## Solutions

**Ex. 1:** The formulas for Euler's method are:

$$\begin{cases} t_0 = 0 \;, \quad y_0^E = 1 \\ t_{n+1} = t_n + 0{,}2 \qquad \text{for } n = 0,1,2,\ldots \\ y_{n+1}^E = y_n^E + h\, f(t_n, y_n^E) = y_n + \frac{1}{5}\left(3 - 5t_n - \frac{1}{2} y_n^E\right) \\ \qquad\qquad\qquad\qquad \text{for } n = 0,1,2,\ldots \end{cases}$$

So
$$t_0 = 0 \;, \quad y_0^E = 1$$
$$t_1 = 0{,}2 \;, \quad y_1^E = 1 + \frac{1}{5}\left(3 - 0 - \frac{1}{2}\right) = 1\tfrac{1}{2}$$
$$t_2 = 0{,}4 \;, \quad y_2^E = 1\tfrac{1}{2} + \frac{1}{5}\left(3 - 1 - \frac{3}{4}\right) = 1\tfrac{3}{4}$$

And $y(0{,}4)$ is approximated by $1\frac{3}{4}$

(For your information: $y(t) = 26 - 10t - 25e^{-\frac{1}{2}t}$ and $y(0{,}4) = 1{,}53$, in two decimal places)

**Ex. 2:** We determine the polar coordinates of this complex number.

$$\left| \frac{(1-i)^8}{(1+i)^6} \right| = \frac{|1-i|^8}{|1+i|^6} = \frac{\sqrt{2}^{\,8}}{\sqrt{2}^{\,6}} = 2$$

$$\arg\left(\frac{(1-i)^8}{(1+i)^6}\right) = 8\arg(1-i) - 6\arg(1+i)$$
$$= 8\left(\frac{-\pi}{4}\right) - 6 \cdot \frac{\pi}{4} = -3\tfrac{1}{2}\pi$$

The corresponding argument in $[0, 2\pi\rangle$ is $\frac{\pi}{2}$,

so $$\frac{(1-i)^8}{(1+i)^6} = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) = 2i$$

**Ex. 3:** $z = e^{\frac{\pi}{4}i} = \frac{1}{2}\sqrt{2} + i\frac{1}{2}\sqrt{2}$, so

$$e^{z} = e^{\frac{1}{2}\sqrt{2} + i\frac{1}{2}\sqrt{2}} = e^{\frac{1}{2}\sqrt{2}}\left(\cos\left(\tfrac{1}{2}\sqrt{2}\right) + i\sin\left(\tfrac{1}{2}\sqrt{2}\right)\right)$$

and $\left|e^{z}\right| = e^{\frac{1}{2}\sqrt{2}}$

**Ex. 4:** Since the given DE is both linear and separable we can use 2 methods to find the answer

Method 1| The DE is linear (written in standard form: $T' + \frac{1}{10}T = 7$) and $e^{\frac{1}{10}t}$ is an integrating factor, so we solve the DE:

$$e^{\frac{1}{10}t}T' + \frac{1}{10}e^{\frac{1}{10}t}T = 7e^{\frac{1}{10}t}$$

$$\Rightarrow \left(e^{\frac{1}{10}t}.T\right)' = 7e^{\frac{1}{10}t}$$

$$\Rightarrow e^{\frac{1}{10}t}.T = 70e^{\frac{1}{10}t} + C$$

where $C \in \mathbb{R}$

$$\Rightarrow T(t) = 70 + C.e^{-\frac{1}{10}t} \text{ where } C \in \mathbb{R}$$

Method 2| The DE is separable so we rewrite the DE as

$$\frac{1}{T-70}\frac{dT}{dt} = -\frac{1}{10} \quad \text{OR} \quad T(t) = 70$$

Integrating both sides with respect to $t$ yields:

$$\int \frac{1}{T-70}\,dT = \int -\frac{1}{10}\,dt \quad \text{OR} \quad T(t) = 70$$

$$\Rightarrow \ln|T-70| = -\frac{1}{10}t + C_1 \text{ where } C_1 \in \mathbb{R}$$

$$\text{OR} \quad T(t) = 70$$

$$\Rightarrow |T-70| = e^{-\frac{1}{10}t + C_1} = e^{C_1} \cdot e^{-\frac{1}{10}t} = C_2 e^{-\frac{1}{10}t}$$

where $C_2 > 0$ OR $T(t) = 70$

$$\Rightarrow T - 70 = C_2 e^{-\frac{1}{10}t} \quad \text{OR} \quad T - 70 = -C_2 e^{-\frac{1}{10}t}$$

where $C_2 > 0$ OR $T(t) = 70$

$$\Rightarrow T(t) = 70 + C e^{-\frac{1}{10}t} \quad \text{where } C \in \mathbb{R}$$

Substituting the initial condition we find $C = 130$, so $T(t) = 70 + 130\, e^{-\frac{1}{10}t}$ °F

Ex. 5: Make the assumption $y_p(t) = a + bt + ct^2 + de^t$ where $a, b, c, d \in \mathbb{R}$.

Then $y_p''(t) = 2c + de^t$ and substituting for $y$ and $y''$ in the DE we obtain

$$2c + de^t + 4a + 4bt + 4ct^2 + 4de^t = t^2 + 3e^t$$

$$\Rightarrow \begin{cases} 2c + 4a = 0 & \Rightarrow a = -\frac{1}{8} \\ 4b = 0 & \Rightarrow b = 0 \\ 4c = 1 & \Rightarrow c = \frac{1}{4} \\ 5d = 3 & \Rightarrow d = \frac{3}{5} \end{cases}$$

So $y_p(t) = -\frac{1}{8} + \frac{1}{4}t^2 + \frac{3}{5}e^t$