Midterm *Linear Algebra* (CSE1205)
11 March 2020, 13:30-15:30, TU Delft

- Calculators and formula sheets are **not** allowed.
- Each question has a unique correct answer.
- Credits: **1 point** for every question.
- Grade: $\max\left(9 \cdot \frac{(P-1)}{12} + 1, 1\right)$, where P denotes the total number of points.

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1. Suppose that A and B are $n \times n$ matrices with $n \geq 2$ such that all the entries $(AB)_{ij}$ of AB are strictly positive. Which of the following matrices must be invertible?

A. None **B.** Only A **C.** Only B **D.** A and B

Answer: A.

2. Determine the last column of the reduced echelon form of $\begin{bmatrix} 0 & 3 & -6 & 6 \\ 3 & -4 & 2 & 1 \\ 3 & -6 & 6 & -3 \end{bmatrix}$.

A. $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ **B.** $\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$ **C.** $\begin{bmatrix} \frac{3}{2} \\ 1 \\ 0 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ **E.** $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ **F.** $\begin{bmatrix} 6 \\ 1 \\ -3 \end{bmatrix}$ **G.** $\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ **H.** $\begin{bmatrix} 0 \\ \frac{2}{3} \\ 1 \end{bmatrix}$

Answer: A.

3. Find the determinant of the matrix $\begin{bmatrix} 4 & 7 & 6 & 2 \\ 0 & 5 & 3 & 0 \\ 2 & 5 & 4 & 1 \\ 8 & 3 & 0 & 1 \end{bmatrix}$.

A. 6 **B.** 4 **C.** 2 **D.** 0 **E.** -2 **F.** -4 **G.** -6 **H.** -8

Answer: G.

4. Suppose the equation $A^T B^{-1}(C^T + X)^T = C$ holds for invertible matrices A, B, C. Solving for X gives that X is equal to:

A. $X = C^T(B^T A^{-1} - I)$ **B.** $X = C^T(A^{-1}B^T - I)$
C. $X = (B^T A^{-1} + I)C^T$ **D.** $X = (A^{-1}B^T + I)C^T$
E. $X = (B^T A^{-1} - I)C^T$ **F.** $X = C^T(A^{-1}B^T + I)$
G. $X = (A^{-1}B^T - I)C^T$ **H.** None of the above

Answer: B.

5. Calculate the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$ if it exists. Then the sum of all the entries of A^{-1} is equal to:

A. -3 **B.** -2 **C.** -1 **D.** 0 **E.** 1 **F.** 2 **G.** 3 **H.** A is not invertible

Answer: D.

6. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation satisfying $T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\2\end{bmatrix}$ and $T\left(\begin{bmatrix}2\\3\\6\end{bmatrix}\right) = \begin{bmatrix}3\\7\end{bmatrix}$.

Calculate $T\left(\begin{bmatrix}0\\3\\4\end{bmatrix}\right)$:

A. $\begin{bmatrix}-1\\-3\end{bmatrix}$ **B.** $\begin{bmatrix}3\\1\end{bmatrix}$ **C.** $\begin{bmatrix}1\\-3\end{bmatrix}$ **D.** $\begin{bmatrix}-3\\1\end{bmatrix}$ **E.** $\begin{bmatrix}-1\\3\end{bmatrix}$ **F.** $\begin{bmatrix}-3\\-1\end{bmatrix}$ **G.** $\begin{bmatrix}1\\3\end{bmatrix}$ **H.** $\begin{bmatrix}3\\-1\end{bmatrix}$

Answer: G.

7. For which values of α is the system $\left[\begin{array}{cccc|c}1 & 3 & -1 & 1 & 1\\3 & 8 & 0 & 1 & 1\\1 & 4 & -4 & \alpha & 4\end{array}\right]$ consistent and has the solution exactly 1 free variable?

A. $\alpha \neq 1$ **B.** $\alpha \neq -1$ **C.** $\alpha \neq 2$ **D.** $\alpha \neq -2$
E. $\alpha \neq 3$ **F.** $\alpha \neq -3$ **G.** $\alpha \neq 4$ **H.** $\alpha \neq -4$

Answer: E.

8. Find the second *row* of the standard matrix of the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first rotates counter clockwise over $\frac{\pi}{6}$ around the origin and then reflects vectors across the x_1-axis:

A. $\left[\frac{1}{2} \quad -\frac{1}{2}\sqrt{3}\right]$ **B.** $\left[-\frac{1}{2} \quad \frac{1}{2}\sqrt{3}\right]$ **C.** $\left[-\frac{1}{2} \quad -\frac{1}{2}\sqrt{3}\right]$ **D.** $\left[\frac{1}{2} \quad \frac{1}{2}\sqrt{3}\right]$
E. $\left[-\frac{1}{2}\sqrt{3} \quad -\frac{1}{2}\right]$ **F.** $\left[-\frac{1}{2}\sqrt{3} \quad \frac{1}{2}\right]$ **G.** $\left[\frac{1}{2}\sqrt{3} \quad -\frac{1}{2}\right]$ **H.** $\left[\frac{1}{2}\sqrt{3} \quad \frac{1}{2}\right]$

Answer: C.

9. For which value of α is the dimension of the span of the following vectors less than 3?

$$\mathbf{v}_1 = \begin{bmatrix}3\\-3\\2\end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix}1\\-2\\1\end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix}-3\\\alpha\\0\end{bmatrix},$$

A. 4 **B.** 3 **C.** 2 **D.** 1 **E.** 0 **F.** -1 **G.** -2 **H.** -3

Answer: H.

10. Let A be a 9×11 matrix with rank 6. What is the dimension of the null space of A?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4 **F.** 5 **G.** 6 **H.** 7

Answer: F.

11. If A is an invertible 4×4 matrix that satisfies the property $A^4(A^{-1})^T = -2A^T$, then find all the possible values for $\det A$:

A. $-4, 4, 0$ **B.** -4 **C.** 4 **D.** $-4, 4$
E. $-\sqrt{2}, \sqrt{2}$ **F.** $\sqrt{2}$ **G.** $-\sqrt{2}\,i, \sqrt{2}\,i$ **H.** 0

Answer: D.

12. Suppose that $A = [\mathbf{a}_1\, \mathbf{a}_2\, \mathbf{a}_3\, \mathbf{a}_4\, \mathbf{a}_5]$ is row equivalent to $\begin{bmatrix}1 & 1 & 1 & 1 & 1\\0 & 1 & 0 & 1 & 1\\0 & 0 & 0 & 1 & 1\\0 & 0 & 0 & 0 & 1\end{bmatrix}$.

Which of the sets $\mathcal{B}_1 = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_4, \mathbf{a}_5\}$ and $\mathcal{B}_2 = \{\mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$ are a basis for $\mathrm{Col}(A)$?

A. none of these **B.** $\mathcal{B}_1$ and $\mathcal{B}_2$ **C.** only $\mathcal{B}_1$ **D.** only $\mathcal{B}_2$

Answer: B.

13. Calculate the coordinate vector $[\mathbf{w}]_{\mathcal{B}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ of the vector $\mathbf{w}$ with respect to the basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, where $\mathbf{w} = \begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$. Then x_2 is given by:

A. -3 **B.** -2 **C.** -1 **D.** 0 **E.** 1 **F.** 2 **G.** 3 **H.** 4

Answer: F.