Resit *Linear Algebra* (AESB1210-15), July 2, 2019, 9:00-12:00

- Calculators and formula sheets are **not** allowed.
- Credits: **4 points** for questions from Part I and **5 points** for questions from Part II.
- The final score: (Total+5)/6, rounded to 1 decimal.
- Write your answers on exam paper.

PART I: SHORT-ANSWER QUESTIONS

1. Solve the following system of equations:

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 6 \\ 3x_1 + 2x_2 + 14x_3 &= 5 \\ x_1 \qquad + 5x_3 &= -2 \end{aligned}$$

2. A is a 3×3-matrix that satisfies the property $A^2 = 2A^T$. What are the possible values for the determinant of A?

3. Let $A = \begin{bmatrix} 4 & 7 & 3 \\ -6 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 6 \\ -5 & 3 & -1 \\ 0 & -4 & -7 \end{bmatrix}$.

a. Write the entries in the first row of AB:

b. Find the entries in the second column of AB:

4. Determine all possible values of h such that the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} -1 \\ h \\ 1 \end{bmatrix}$ are linearly independent. (Write **NOT** if there are no such h):

5. Find the LU-decomposition $A = LU$ of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -3 & -10 & 2 \end{bmatrix}$:

$L =$

$U =$

6. For the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ it is given that $T(\mathbf{e}_1 + \mathbf{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and

$T(\mathbf{e}_1 - \mathbf{e}_2) = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$. Find the standard matrix A of T:

7. Find the orthogonal projection of $\mathbf{y} = \begin{bmatrix} 18 \\ 12 \\ 6 \end{bmatrix}$ onto $W = \text{Span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$:

$\hat{\mathbf{y}} =$

8. Consider the matrix $A = \begin{bmatrix} 3 & -4 & 7 \\ 4 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.

a. Find the characteristic polynomial $p(\lambda)$ of A:

b. Find all the (real, and possibly, complex) eigenvalues of A:

9. Consider the overdetermined system $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 0 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$.

a. Give the augmented matrix of the normal equations that you have to solve to find the least-squares solution of this system:

b. Find all the least-squares solutions:

10. **a.** Find the dimension of $W := \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix} \right\}$ in $\mathbb{R}^4$: $\dim W =$

b. Find a basis for $W^{\perp}$:

END OF PART I. GO TO PART II (OPEN QUESTIONS)!

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PART II: OPEN QUESTIONS

- Justify clearly your answer.
- Mention the theorems, corollaries and results you are using!

11. **a.** Find the reduced echelon form of the matrix $A := \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$

b. Find the rank of A.

c. Find the dimension of $\mathrm{Nul}(A)$.

d. Find a basis for $\mathrm{Col}(A)$.

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12. **a.** Calculate the determinant of the matrix $A := \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & a \end{bmatrix}$.

b. Find all values of a such that A is invertible.

c. Find the inverse A^{-1} for $a = 1$.

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13. **a.** Find the eigenvalues of $A := \begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix}$.

b. Find a basis for all the eigenspaces E_λ.

c. Give a diagonalization of A (if it exists), i.e. write A in the form PDP^{-1} with D a diagonal matrix.