Resit Linear Algebra (AESB1210-15), July 2, 2019, 9:00-12:00

• Calculators and formula sheets are **not** allowed.

• Credits: 4 points for questions from Part I and 5 points for questions from Part II.

• The final score: (Total+5)/6, rounded to 1 decimal.

• Write your answers on exam paper.

PART I: SHORT-ANSWER QUESTIONS

1. Solve the following system of equations:

$$x_1 + x_2 + 2x_3 = 6$$

$$3x_1 + 2x_2 + 14x_3 = 5$$

$$x_1 + 5x_3 = -2$$

2. A is a 3×3 -matrix that satisfies the property $A^2 = 2A^T$. What are the possible values for the determinant of A?

3. Let $A = \begin{bmatrix} 4 & 7 & 3 \\ -6 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 6 \\ -5 & 3 & -1 \\ 0 & -4 & -7 \end{bmatrix}$.

a. Write the entries in the first row of AB:

b. Find the entries in the second column of AB:



4. Determine all possible values of h such that the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} -1 \\ h \\ 1 \end{bmatrix}$ are linearly independent. (Write **NOT** if there are no such h):

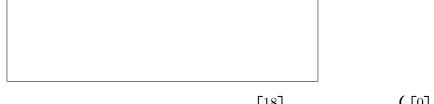


5. Find the LU-decomposition A = LU of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -3 & -10 & 2 \end{bmatrix}$:

$$L=igsqcut U=igsqcut U$$

6. For the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ it is given that $T(\mathbf{e}_1 + \mathbf{e}_2) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and

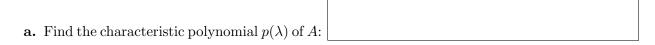
$$T(\mathbf{e}_1 - \mathbf{e}_2) = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$
. Find the standard matrix A of T :



7. Find the orthogonal projection of $\mathbf{y} = \begin{bmatrix} 18\\12\\6 \end{bmatrix}$ onto $W = \operatorname{Span} \left\{ \begin{bmatrix} 0\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\1\\-1 \end{bmatrix} \right\}$:

$$\hat{\mathbf{y}} =$$

8. Consider the matrix $A = \begin{bmatrix} 3 & -4 & 7 \\ 4 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.



b. Find all the (real, and possibly, complex) eigenvalues of A:

9. Consider the overdetermined system $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 0 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$.

a. Give the augmented matrix of the normal equations that you have to solve to find the least-squares solution of this system:

b. Find all the least-squares solutions:

10. a. Find the dimension of $W := \operatorname{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix} \right\}$ in \mathbb{R}^4 : dim $W = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix}$

b. Find a basis for W^{\perp} :

END OF PART I. GO TO PART II (OPEN QUESTIONS)!

(continued on next page!)

PART II: OPEN QUESTIONS

•	Justify	clearly	vour	answer.

• Mention the theorems, corollaries and results you are using!

		I	3	1	-2	-3
11	Tind the reduced scholar forms of the matrix A.	1	4	3	-1	-4
11.	a. Find the reduced echelon form of the matrix $A :=$	2	3	-4	-7	-3
	a. Find the reduced echelon form of the matrix $A :=$ b. Find the rank of A .	$ _3$	8	1	-7	-8

c. Find the dimension of Nul(A).

d.	Find	a.	hasis	for	Col	(A)

(continued on next page!)

		I	1	0	
	Calculate the determinant of the matrix $A :=$				
b.	Find all values of a such that A is invertible.	1	1	a	

c. Find the inverse A^{-1} for a=1.

C. I'ma the inverse A	$101 \ \alpha = 1.$	

(continued on next page!)

- **13. a.** Find the eigenvalues of $A := \begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix}$.
 - **b.** Find a basis for all the eigenspaces E_{λ} .
 - **c.** Give a diagonalization of A (if it exists), i.e. write A in the form PDP^{-1} with D a diagonal matrix.