Delft University of Technology, EEMCS faculty
Examination Mathematics 2, AESB1210-15
Thursday, April 12th, 2018, 9.00-12.00

- It's not allowed to use a calculator or a mathematical table.
- Each answer should be clearly motivated.
- Simplify your answer as much as possible.
- Your grade is obtained by rounding (score+5)/5 to the nearest half.
- Points:

Ex. 1	5,5	Ex. 2	3	Ex. 3	3,5	Ex. 4	3	Ex. 5	3
Ex. 6a	4	Ex. 7	3	Ex. 8a	2,5	Ex. 9a	4,5	Ex. 10	4
Ex. 6b	2			Ex. 8b	2,5	Ex. 9b	2		
				Ex. 8c	2,5				

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1. Find the general solution, in explicit form, of $e^x y' + x\sqrt{4-y} = 0$.

2. Let $y(t)$ be the solution of the initial value problem $y' - 2y = 0.5 - t$ and $y(0) = 1$. Use Euler's method with step size $h = 0.5$ to approximate $y(1)$.

3. Make the substitution $v(x) = e^{2y(x)}$ to convert $2xe^{2y}\dfrac{dy}{dx} = 3x^4 + e^{2y}$, with $x > 0$, into a linear differential equation. Write this linear differential equation in standard form (*don't solve this linear differential equation*).

4. Find $|e^{e^z}|$ if $z = \ln(8) + \frac{1}{4}\pi i$.

5. Find a trial solution for the differential equation $4y'' - 12y' + 9y = 24xe^{\frac{3}{2}x}$ if the method of undetermined coefficients is used. *Do not determine the coefficients.*

6. The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3\alpha \\ 7 & 2\alpha + 14 & 43 - 7\alpha \end{bmatrix}, \text{ where } \alpha \in \mathbb{R}.$$

a. For which value(s) of α is vector $\begin{bmatrix} 4 \\ 7 \\ 40 \end{bmatrix}$ in the range of T?

b. For which value(s) of α is vector $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ in $NUL(A)$?

P.T.O.

7. Find matrix B if $\left(B^T - 3\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$.

8. Mark each statement True or False, justify your answers.

   a. **Statement 1**: If $\underline{a}, \underline{b} \in \mathbb{R}^n$ and $\{\underline{a}+\underline{b}, \underline{a}-\underline{b}\}$ is linearly independent then $\{\underline{a}, \underline{b}\}$ is linearly independent.

   b. **Statement 2**: If $n \times n$ matrices E and F have the property that $EF = I_n$, the $n \times n$ identity matrix, then $EF = FE$.

   c. **Statement 3**: If $n \times n$ matrices A and B have the property that $rank(A) = rank(B)$ then $rank(A^2) = rank(B^2)$.

9. Let $\underline{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ and $U = Span\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 1 \\ 1 \end{bmatrix}\right\}$.

   a. Write $\underline{x}$ as the sum of a vector $\underline{u} \in U$ and a vector $\underline{w} \in U^{\perp}$, the orthogonal complement of U in $\mathbb{R}^4$.

   b. Find a basis for $U^{\perp}$. → GS

10. A certain experiment generates the data $(0, -\frac{1}{2})$, $(\frac{1}{4}\pi, \frac{1}{2}\sqrt{2})$, $(\frac{1}{2}\pi, 1)$ and $(\frac{3}{4}\pi, \frac{1}{2}\sqrt{2})$ in the $xy-$plane. Find the least-squares fit of the form $y = \alpha\cos(x) + \beta\sin(x)$, where $\alpha, \beta \in \mathbb{R}$.