toets 3.

10). $\int_0^e x^9 \ln x \, dx$. Merk op: $x^9 \ln x$ is niet gedefinieerd voor $x=0$.
wel geldt $\lim_{x \to 0^+} x^9 \ln x = 0$ zodat
de integraal niet echt oneigenlijk is.

$$\int x^9 \ln x \, dx = \int \ln x \, d\tfrac{1}{10}x^{10} = \tfrac{1}{10}x^{10}\ln x - \int \tfrac{1}{10}x^{10} \cdot \tfrac{1}{x}\, dx$$

$$= \tfrac{1}{10}x^{10}\ln x - \int \tfrac{1}{10}x^9 dx = \tfrac{1}{10}x^{10}\ln x - \tfrac{1}{100}x^{10}.$$

$$\int_0^e x^9 \ln x \, dx = \lim_{p \to 0^+} \left[\tfrac{1}{10}x^{10}\ln x - \tfrac{1}{100}x^{10}\right]_p^e =$$

$$\tfrac{1}{10}e^{10} - \tfrac{1}{100}e^{10} = \tfrac{9}{100}e^{10}$$

11) $\int_0^1 \frac{x^3+x^2}{x^2+1} dx$.

staartdeling: $x^2+1 / x^3+x^2 \backslash x+1$

$$\begin{array}{l} x^3+x^2 \\ \underline{x^3+x} \\ x^2-x \\ \underline{x^2+1} \\ -x-1. \end{array}$$

$$\int_0^1 \left(x+1 - \frac{x+1}{x^2+1}\right) dx = \int_0^1 \left(x+1 - \frac{x}{x^2+1} - \frac{1}{x^2+1}\right) dx =$$

$$\left[\tfrac{1}{2}x^2 + x - \tfrac{1}{2}\ln(x^2+1) - \arctan x\right]_0^1 = \tfrac{3}{2} - \tfrac{1}{2}\ln 2 - \tfrac{\pi}{4}$$

12). a) $0 \le \frac{1}{\sqrt{x}(1+x)} \le \frac{1}{\sqrt{x}}$ $\int_0^1 \frac{1}{\sqrt{x}} dx$ is convergent en ($p = \frac{1}{2} < 1$)

dus ook: $\int_0^1 \frac{1}{\sqrt{x}(1+x)} dx$ is convergent.

b). $\int_0^\infty \frac{1}{\sqrt{x}(1+x)} dx = \int_0^1 \frac{dx}{\sqrt{x}(1+x)} + \int_1^\infty \frac{dx}{\sqrt{x}(1+x)}$.

12.b vervolg: $\int_0^1 \frac{dx}{\sqrt{x}(1+x)}$ is convergent, zie (a)

Nu nog $\int_1^\infty \frac{dx}{\sqrt{x}(1+x)}$. $\quad 0 \le \frac{1}{\sqrt{x}(1+x)} \le \frac{1}{x\sqrt{x}}$

$\int_1^\infty \frac{1}{x\sqrt{x}}\,dx$ is convergent $(p = \frac{3}{2} > 1)$, dus

ook $\int_1^\infty \frac{dx}{\sqrt{x}(1+x)}$ is convergent.

Conclusie $\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx$ is convergent.

<u>12. Alternatief</u>: Bepaal eerst een primitieve

$$\int \frac{1}{\sqrt{x}(1+x)}\,dx \underset{\uparrow}{=} \int \frac{1}{t(1+t^2)}\cdot 2t\,dt = \int \frac{2}{1+t^2}\,dt =$$

$$\boxed{\begin{array}{c} \sqrt{x} = t \\ x = t^2 \\ dx = 2t\,dt \end{array}} \qquad 2\arctan t + C = 2\arctan\sqrt{x} + C.$$

a) $\int_0^1 \frac{1}{\sqrt{x}(1+x)}\,dx = \lim_{a \to 0^+} \Big[2\arctan\sqrt{x}\Big]_a^1 = \frac{\pi}{2}$,

dus convergent.

b) $\int_1^\infty \frac{1}{\sqrt{x}(1+x)}\,dx = \lim_{p \to \infty} \Big[2\arctan\sqrt{x}\Big]_1^p = \pi - \frac{\pi}{2} = \frac{\pi}{2}$.

en $\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx = \frac{\pi}{2} + \frac{\pi}{2} = \pi$ en dus ook convergent.

13\. a) $\underline{r}(1) = \langle -3, 0, 2\pi \rangle$, een raakvector aan de kromme in $(-3, 0, 2\pi)$ is dus $\underline{r}'(1)$.

$\underline{r}'(t) = \langle -3\pi \sin \pi t,\ 3\pi \cos \pi t,\ 3\pi \sqrt{t} \rangle$.

$\underline{r}'(1) = \langle 0, -3\pi, 3\pi \rangle$.

b).
$$\int_0^3 |\underline{r}'(t)|\,dt = \int_0^3 \sqrt{9\pi^2 \sin^2 \pi t + 9\pi^2 \cos^2 \pi t + 9\pi^2 t}\,dt$$

$$= \int_0^3 \sqrt{9\pi^2 + 9\pi^2 t}\,dt = 3\pi \int_0^3 \sqrt{1+t}\,dt =$$

$$\left[ 2\pi (1+t)^{\frac{3}{2}} \right]_0^3 = 2\pi (8-1) = 14\pi.$$