toets 2 27 november 2013

5. a) $f(x) = x^3$ $\quad df = 3x^2 dx = 3 \cdot 900 \cdot \frac{1}{10} = 270.$

b) $\frac{dx}{x} = \frac{0.1}{30} = \frac{1}{300}$

$\frac{df}{f} = \frac{270}{(30)^3} = \frac{270}{27000} = \frac{1}{100}$

6. $f(x) = \tan x$ $\quad f(\frac{\pi}{4}) = 1$

$f'(x) = \frac{1}{\cos^2 x} = (\cos x)^{-2}$ $\quad f'(\frac{\pi}{4}) = 2$

$f''(x) = -2(\cos x)^{-3} \cdot (-\sin x) = \frac{2\sin x}{\cos^3 x}$ $\quad f''(\frac{\pi}{4}) = 4.$

$f'''(x) = \frac{2\cos^4 x - 3\cos^2 x(-\sin x)2\sin x}{\cos^6 x}$

$= \frac{2\cos^2 x + 6\sin^2 x}{\cos^4 x}$ $\quad f'''(\frac{\pi}{4}) = \frac{1+3}{\frac{1}{4}} = 16.$

$T_3(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{3!}f'''(a)(x-a)^3$

$= 1 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2 + \frac{8}{3}(x - \frac{\pi}{4})^3.$

7. $a_1 = 3 \quad a_2 = r a_1 \Leftrightarrow 3r < 0 \rightarrow r < 0.$

$\frac{1}{3} = a_3 = r a_2 = r^2 a_1 = r^2 \cdot 3 \rightarrow r^2 = \frac{1}{9}$ en $r = -\frac{1}{3}.$

$|r| = |-\frac{1}{3}| = \frac{1}{3} < 1$ dus de reeks is convergent.

met som $\frac{3}{1-(-\frac{1}{3})} = \frac{3}{1+\frac{1}{3}} = \frac{9}{4}$

8. $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$x = 0 \rightarrow t = 0$

$x = 2 \rightarrow t = \sqrt{2}$

$$\int_0^{\sqrt{2}} \frac{e^t}{\sqrt{t^2+1}} \cdot 2t\,dt.$$

g. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\,dx = -\int_{x=0}^{\frac{\pi}{2}} \frac{1}{1+\cos^2 x}\,d\cos x =$

$\cos x = t$

$x = 0 \to t = 1$

$x = \frac{\pi}{2} \to t = 0$

$-\int_1^0 \frac{1}{1+t^2}\,dt = \left[-\arctan t\right]_1^0 = \frac{\pi}{4}.$