AESB1110, toets 3, 4 november 2013.

1. $\int_0^{\frac{1}{2}} (x+1)\sin(\pi x)\,dx = -\frac{1}{\pi}\int_{x=0}^{\frac{1}{2}} (x+1)\,d\cos\pi x =$

$\left[-\frac{1}{\pi}(x+1)\cos(\pi x)\right]_0^{\frac{1}{2}} + \frac{1}{\pi}\int_0^{\frac{1}{2}} \cos(\pi x)\,dx = \frac{1}{\pi} + \frac{1}{\pi^2}\left[\sin(\pi x)\right]_0^{\frac{1}{2}}$

$= \frac{1}{\pi} + \frac{1}{\pi^2}$

2. $\int_0^1 \ln(1+x^2)\,dx = \left[x\ln(1+x^2)\right]_0^1 - \int_0^1 \frac{2x^2}{1+x^2}\,dx =$

$\ln 2 - 2\int_0^1 \frac{x^2+1-1}{1+x^2}\,dx \underset{\uparrow}{=} \ln 2 - 2\int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx =$

afstaartdeling.

$\ln 2 - 2\left[x - \arctan x\right]_0^1 = \ln 2 - 2\left(1 - \frac{\pi}{4}\right) = \ln 2 - 2 + \frac{\pi}{2}$

3 a) $\int \frac{1}{x^2+4x+5}\,dx = \int \frac{1}{(x+2)^2+1}\,dx = \arctan(x+2) + C$

b) $\int_1^\infty \frac{1}{x^2+4x+5}\,dx = \lim_{p\to\infty}\left[\arctan(x+2)\right]_1^p = \frac{\pi}{2} - \arctan 3$

dus convergent.

of $0 \le \frac{1}{x^2+4x+5} \le \frac{1}{x^2}$. $\int_1^\infty \frac{1}{x^2}\,dx$ is convergent dus

is ook $\int_1^\infty \frac{1}{x^2+4x+5}\,dx$ convergent

4. Breuksplitsen.

$\frac{x+2}{x^2-1} = \frac{x+2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

$A(x+1) + B(x-1) = x+2$ voor iedere x.

$(A+B)x + A - B = x + 2$ $\quad \begin{cases} A+B=1 \\ A-B=2 \end{cases}$ $\quad A = \frac{3}{2}$, $B = -\frac{1}{2}$

$\int \frac{x+2}{x^2-1}\,dx = \int\left(\frac{3}{2}\frac{1}{x-1} - \frac{1}{2}\frac{1}{x+1}\right)dx = \frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$

5a) $\underline{r}(t) = \langle \sqrt{8t}, t, \ln t \rangle$, $t > 0$, $\underline{r}(2) = \langle 4, 2, \ln 2 \rangle$

$$\underline{r}'(t) = \left\langle \frac{8}{2\sqrt{8t}}, 1, \frac{1}{t} \right\rangle$$

$\underline{r}'(2) = \langle 1, 1, \frac{1}{2} \rangle$, $|\underline{r}'(2)| = \sqrt{2\frac{1}{4}} = \frac{3}{2}$

Dus de eenheidsraakvector is $\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

b) $$\int_1^3 |\underline{r}'(t)|\,dt = \int_1^3 \sqrt{\frac{64}{32t} + 1 + \frac{1}{t^2}}\,dt = \int_1^3 \sqrt{\frac{2t + t^2 + 1}{t^2}}\,dt$$

$$= \int_1^3 \frac{t+1}{t}\,dt = \int_1^3 \left(1 + \frac{1}{t}\right)dt = \Big[t + \ln|t|\Big]_1^3 = 3 + \ln 3 - 1$$

$$= 2 + \ln 3.$$