

Mathematics 1 AESB1110: Exam 1

September 26, 2014

Rules: Write clearly and provide complete derivations/explanations

Questions:

1. Given the points $A(1, 1, 3)$, $B(3, 1, 1)$, and $C(1, 3, 1)$:

- (a) Find the angle $\angle ABC$ 1 p.
- (b) Find the area of the triangle ABC 1 p.
- (c) Give the equation of the plane passing through these points 1 p.
- (d) Consider the plane passing through the point A and having the normal vector \underline{AB} . Find the angle between this plane and the plane constructed in (c)? 1 p.

2. Simplify : 1 p.

$$f(x) = \sin(\cos^{-1}(x))$$

3. Given the function : 1 p.

$$g(x) = \begin{cases} x^3 - 1, & x < 2, \\ a, & x = 2, \\ \frac{x^2 + 3x - 10}{x - 2}, & x > 2. \end{cases}$$

find the value of the constant a for which $g(x)$ is continuous on \mathbb{R} .

4. Evaluate the following two limits:

$$\lim_{x \rightarrow 0} x^4 \sin\left(\frac{1}{x}\right), \quad \text{and} \quad \lim_{x \rightarrow 1} \frac{\sin(1-x)}{x-1}$$

2 p.

5. (a) Find y' from the following implicit definition of $y(x)$: 1 p.

$$e^y \cos(x) = 1 + \sin(xy)$$

(b) Prove that 1 p.

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$