Uitwerking toets 2 AESB1110, 18 oktober 2013.

1. a) $r = 30$ met een maximale fout van $dr = 0.05$

De oppervlakte $f(r) = \pi r^2$

$d f(r) = f'(r) dr = 2\pi r \, dr$

Met $r = 30$ en $dr = 0.05$: $\quad d f(r) = 2\pi \cdot 30 \cdot 0.05 = 3\pi$

b) De relatieve fout $\dfrac{d f(r)}{f(r)} = \dfrac{3\pi}{\pi\, 30^2} = \dfrac{1}{300} \approx 0.33\ \%$

2a) m.b.v. formuleblad:

$e^x = 1 + x + \frac{1}{2}x^2 + \dots$

$e^{-x} = 1 - x + \frac{1}{2}x^2 - \dots$

$e^{-x^2} = 1 - x^2 + \frac{1}{2}x^4 - \dots$

$T_4(x) = 1 - x^2 + \frac{1}{2}x^4$

of: $T_4(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + \frac{1}{6} f'''(0)x^3 + \frac{1}{24} f^{(4)}(0)x^4$.

$f(x) = e^{-x^2}$

$f'(x) = -2x e^{-x^2}$

$f''(x) = -2e^{-x^2} + 4x^2 e^{-x^2}$

$f'''(x) = 4x e^{-x^2} + 8x e^{-x^2} - 8x^3 e^{-x^2}$

$= 12x e^{-x^2} - 8x^3 e^{-x^2}$

$f^{(4)}(x) = 12 e^{-x^2} - 24x^2 e^{-x^2} - 24x^2 e^{-x^2} + 16x^4 e^{-x^2}$

$f(0) = 1, \ f'(0) = 0, \ f''(0) = -2, \ f'''(0) = 0, \ f^{(4)}(0) = 12$

$T_4(x) = 1 - x^2 + \frac{1}{24} \cdot 12x^4 = 1 - x^2 + \frac{1}{2}x^4$.

b) los op met GR: $|f(x) - T_4(x)| < 0.05$

$\left| e^{-x^2} - (1 - x^2 + \frac{1}{2}x^4) \right| < 0.05$

$-0.84141 < x < 0.84141$

3. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+2}} = \sum_{n=0}^{\infty} \frac{1}{9}\left(-\frac{1}{3}\right)^n$. De eerste term $a = \frac{1}{9}$ en $r = -\frac{1}{3}$.

$|r| = \frac{1}{3} < 1$ dus de reeks is convergent met som

$$\frac{a}{1-r} = \frac{\frac{1}{9}}{1 + \frac{1}{3}} = \frac{1}{12}$$

4. $$\int_0^2 \frac{x^2}{\sqrt{x^3+1}}\,dx = \int_1^9 \frac{1}{\sqrt{u}}\,\frac{1}{3}\,du = \int_1^9 \frac{1}{3}u^{-\frac{1}{2}}\,du = \left[\frac{2}{3}u^{\frac{1}{2}}\right]_1^9$$

$$\boxed{\begin{array}{l} x^3+1=u \\ 3x^2\,dx=du \\ x=0 \to u=1 \\ x=2 \to u=9 \end{array}}$$

$$= \frac{2}{3}(3-1) = \frac{4}{3}$$

5. $$\int_0^{\pi/6} \cos^3 x\,dx = \int_{x=0}^{\pi/6} \cos^2 x\,d\sin x = \int_{x=0}^{\pi/6} (1-\sin^2 x)\,d\sin x =$$

$$\boxed{\begin{array}{l} \sin x = u \\ x=0 \to u=0 \\ x=\frac{\pi}{6} \to u=\frac{1}{2} \end{array}}$$

$$\int_0^{1/2} (1-u^2)\,du = \left[u-\frac{1}{3}u^3\right]_0^{1/2}$$

$$= \frac{1}{2} - \frac{1}{24} = \frac{11}{24}.$$

6. $$\int \frac{1}{e^x+e^{-x}}\,dx = \int \frac{1}{u+\frac{1}{u}}\cdot\frac{1}{u}\,du = \int \frac{1}{u^2+1}\,du$$

$$\boxed{\begin{array}{l} e^x=u \\ x=\ln u \\ dx=\frac{1}{u}\,du \end{array}}$$

$$= \arctan u + C$$

$$= \arctan(e^x) + C.$$