Exam April 11 2019 9u00-11u30

Analyse 2 CTB1001-16 Mathematics 1 AESB1211

During this exam it is not allowed to use a calculator or any other electronic device. You can use the Analysis Formula Sheet. The exam consists of 14 questions. For the short-answer questions you only have to give the answer to the question, no calculations. The solutions to the open questions on the other hand have to be presented in full. Explain the various steps you take in your solution.

Name:

Student number:

Short-answer questions Each correct answer is good for one point.

Question 1. Determine the point of intersection of the line through the points $(1, 2, 0)$ and $(0, 3, 1)$ and the plane $2x - 4y + z = 4$.

$$(3, 0, -2)$$

Question 2. The function $f(x, y)$ satisfies the following properties: $f(2, 3) = 4$, $f_x(2, 3) = -2$ and $f_y(2, 3) = -1$. Give an approximation of the value $f(1.8, 3.1)$ by using a linearisation in the point $(2, 3)$.

$$f(1.8, 3.1) \approx 4.3$$

Question 3. Determine $\dfrac{\partial z}{\partial y}$ if the function $z(x, y)$ is implicitely defined by the equation

$$x^2 + xy + y^2 + 3xz + z^2 = 1.$$

$$-\frac{x + 2y}{3x + 2z}$$

Question 4. For a certain function $f(x, y, z)$ the length of the vector $\nabla f(1, -1, 0)$ is equal to 6. The angle between the unit vector $\mathbf{u}$ and the vector $\nabla f(1, -1, 0)$ is equal to $\dfrac{\pi}{3}$. What is the value of the directional derivative $D_{\mathrm{u}} f(1, -1, 0)$?

$$3$$

Question 5. Evaluate the following integral. Write your answer as a single fraction

$$\iint_R \frac{1}{(1+x+y)^3}\, dA \qquad R = [1,3]\times[0,2]$$

$$\frac{1}{12}$$

Question 6. R is the area in the second quadrant of the xy-plane between the circles $x^2+y^2=1$ and $x^2+y^2=4$. Evaluate the following integral.

$$\iint_R y\, dA$$

$$\frac{7}{3}$$

Question 7. Determine the value $\dfrac{\partial^2 f}{\partial x \partial y}(3,-1)$ when $f(x,y)$ is equal to $\sqrt{x+y^2}$.

$$\frac{1}{16}$$

Question 8. Determine the directional derivative of the function $f(x,y) = xy + y^2$ in the point $(2,1)$ in the direction of the vector $\langle 1,-1\rangle$.

$$-\frac{3}{\sqrt{2}} \text{ of } -\frac{3\sqrt{2}}{2}$$

Question 9. Rewrite the following integral as an integral in cylindrical coordinates. You don't have to compute the value of the integral.

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^{1-x^2-y^2} x\, dz\, dy\, dx$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^1\int_0^{1-r^2} r^2\cos(\theta)\, dz\, dr\, d\theta$$

Question 10. Determine an equation of the tangent plane in the point $(2, -1, 2)$ to the graph of the function $z = \dfrac{3+y}{x+y}$.

$$z = -2x - y + 5$$

Question 11. Determine an equation of the plane that contains the point $(3, 1, 1)$ and that is perpendicular to the line through the points $(2, 0, 2)$ and $(1, 1, 1)$.

$$x - y + z = 3$$

Open questions

Question 12. (3 points) The function $f(x, y)$ is defined as $x^2 + x - 3xy + y^3$.

1. Give all critical points of the function $f(x, y)$.
2. Determine for each of these points whether it is a saddle point or if it is a point in which the function attains a local minimum or maximum.

Answer: The critical points are $(1, 1)$ and $(1/4, 1/2)$. The point $(1, 1)$ is a local minmum and $(1/4, 1/2)$ is a saddle point.

Question 13. (3 points) R is the area in the xy-plane between the parabola $y = x^2 + 1$ and the line $y = x + 3$. Assume that the density in each point is equal to the constant ρ. Determine the x-coordinate of the center of mass of R.

Answer: The mass is equal to

$$\int_{-1}^{2} \int_{x^2+1}^{x+3} \rho \; dy \; dx = \frac{9\rho}{2}.$$

The x-coordinate is

$$\frac{2}{9\rho} \int_{-1}^{2} \int_{x^2+1}^{x+3} x\rho \; dy \; dx = \frac{1}{2}.$$

Question 14. (3 points) E is the solid that consists of all points (x, y, z) within the sphere centred at the origin and with radius 2 that satisfy the conditions $x \geq 0$ and $z \leq 0$. Evaluate the following integral.

$$\iiint_E 5z^2 \, dV$$

Answer: In spherical coordinates this becomes

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_0^2 5\rho^4 \cos^2(\phi) \sin(\phi) \; d\rho \; d\phi \; d\theta.$$

This is equal to $\dfrac{32\pi}{3}$.