Mass Transfer

VI. Unsteady and Multivariate Diffusion

A. **Conditions for analogy between unsteady diffusion and unsteady conduction**

If:

- no convection, no generation (no chemical reactions) within region of interest
- uniform and constant D_{AB}
- simplified version of Fick's law applies (dilute solutions)

... then tabulated solutions unsteady and multivariate heat conduction apply to unsteady diffusion processes

1. Note: principles for "extending 1D solutions" apply as well
   - surface of zero flux (no mass transfer across surface)
   - product method for orthogonal conduction
   - superposition

Principles for dealing with "complex" problems apply, too.

B. **correspondence between variables in conduction and diffusion processes**

<u>heat conduction</u>		<u>diffusion</u>
T, T_1, T_0	$\longrightarrow$	C_A, C_{A0}, C_{A1} (or x_A, x_{A0}, x_{A1}, or $\omega_A, \omega_{A0}, \omega_{A1}$)
$\alpha = \frac{k}{\rho c_p}$	$\longrightarrow$	D_{AB} (units m^2/s)
B, R, t, D, r, y	$\longrightarrow$	unchanged
$\frac{\vec{q}}{\rho c_p}$	$\longrightarrow$	$\vec{N}_A$

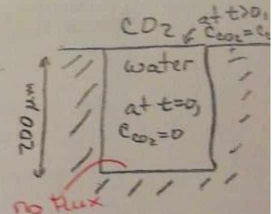
C. **examples**

$Pr \longrightarrow Sc$ "Schmidt number"

$Nu \longrightarrow Sh$ "Sherwood number"

Examples of Unsteady Diffusion

1. Diffusion of CO_2 into water-filled pore. How long until water at wall at far end of pore is 90% saturated with CO_2 (i.e., $c_{CO_2} = 0.9\, c_{CO_2,1}$ at far wall).



Assume CO_2 is "dilute" in aqueous solution. Ignore possible reactions with rock. Assume $\mathcal{D}_{CO_2,H_2O} = \text{const.} \cong 2\cdot 10^{-10}\ m^2/s$

<u>Solution:</u> no-flux condition at far wall is equivalent to an insulated boundary in heat conduction. Thus the geomet[ry] is equivalent to a 400 μm-wide slab exposed to CO_2 on both sides. $b = 200\,\mu m = 2\cdot 10^{-4}\ m$. We want time for

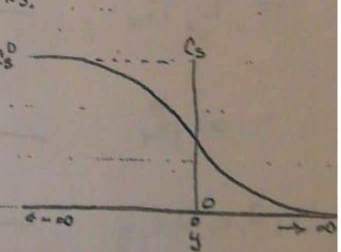
$$\frac{c_{CO_2} - c_{CO_2,0}}{c_{CO_2,1} - c_{CO_2,0}} = \frac{c_{CO_2}}{c_{CO_2,1}} = 0.9 \longleftrightarrow \frac{T - T_0}{T_1 - T_0} = 0.9 \text{ at } y/b = 0.$$

From Fig 12.1-1, $\frac{\alpha t}{b^2} \approx 1.0 \leftrightarrow \frac{\mathcal{D} t}{b^2} = 1.0$
BSLK 11.5-1

$b = 200$ sec.

Diffusion between semi-infinite blocks.

Identical porous media, one saturated with dilute brine, one with pure water, are brought into contact. There is diffusion of salt, but no bulk flow. Assume $\mathcal{D}_s \approx 5\cdot 10^{-10}\ m^2/s$.



a) How far into saline block has c_s fallen to $0.9\, c_s^0$ after

Want $\frac{c_s - c_{sb}}{c_{sf} - c_{sb}} = \frac{0.9\, c_s^0 - 0}{c_s^0 - 0} = 0.9$. From figure for unsteady conduction between semi-infinite blocks in contact,

$$y/\sqrt{4\alpha t} \leftrightarrow \frac{y}{\sqrt{4\mathcal{D} t}} \cong 0.92 \rightarrow y = (0.92)\left[(4)(5\cdot 10^{-10})(8.64\cdot 10^4)\right]^{1/2} = ...$$