VII. ~~[illegible]~~. **Mass-transfer coefficients**

A. **review of effects of turbulence**

- accelerates rate of mass transfer
- too complex for analytical solution
- correlate mass-transfer rate using dimensionless groups

B. **Mass transfer in tubes**

1. **definition of mass-transfer coefficient $k_{x,ln}$**

Methods described here apply only if mass transfer rates are "low" and "simplified Fick's law" applies. ~~In that case, BSL Eq. [illegible] is~~ ~~[illegible]~~ Directly analogous to Fourier's law.

$$\mathcal{W}_A^{(m)} = k_{x,ln}\,(\pi D L)\,(x_{A0} - x_{AB})_{ln} \qquad (\mathrm{I}_m)$$

total rate of transport of solute A into fluid from wall

mass transfer coefficient

area

log average difference in x_A between fluid at wall and fluid in bulk (i.e., from away from wall)

(~~[illegible] for heat transfer~~)

Note: C_{A0} is C_A *in fluid next to wall* (typically in equilibrium with solid at wall), not C_A in solid wall itself

2. **mass-conservation equation (Eq. "IIm")**

$$\mathcal{W}_A^{(m)} = \underbrace{(C_{Ab2} - C_{Ab1})\,\pi R^2 \langle v \rangle}_{} \qquad (\mathrm{II}_m)$$

mass transfer rate from wall

increase in A in fluid as fluid flows through tube

(similar to "eq. II" in notes on heat transfer in tubes; derivation is similar)

Combine eqs. "Im" and "IIm" to derive Eq. "IIIm"

$$\frac{k_{x,ln} D}{c\,\mathcal{D}_{AB}} = Sh = \ln\left[\frac{C_{A0} - C_{Ab1}}{C_{A0} - C_{Ab2}}\right] Re\,Sc\,\frac{D}{4L} \qquad (\mathrm{III}_m)$$

concentration of fluid entering

Sherwood number - formerly called Nu_{AB}, Nusselt # for mass transfer

surface concentration of A

concentration of fluid leaving

Schmidt #

3. analogy between heat- and mass-transfer coefficients in tubes

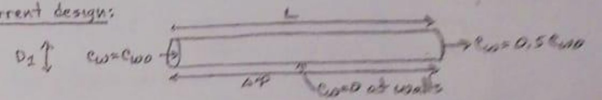
For cases in which "simplified Fick's law" applies to mass transfer in tubes, one can use the correlations for heat transfer after making the following substitutions:

heat transfer		mass transfer
$\cancel{T}\ T, T_0, T_b, \ldots$	$\rightarrow$	$c_A, c_{A0}, c_{Ab}, \ldots$ (or x_A, x_{A0}, x_{Ab} or $\omega_A, \omega_{A0}, \omega_{Ab}, \ldots$)
$Pr = \frac{\hat{C}_p \mu}{k} = \frac{\cancel{\rho}\nu}{\alpha}$	$\rightarrow$	$Sc = \frac{\mu}{\rho \cdot \mathcal{D}_{AB}} = \frac{\nu}{\mathcal{D}_{AB}}$ "Schmidt #"
$Nu = \frac{h_{ln} D}{k}$	$\rightarrow$	$Sh = \frac{k_{x,ln} D}{c \mathcal{D}_{AB}}$ "Sherwood #"
Eq. III_{h}	$\rightarrow$	Eq. III_m

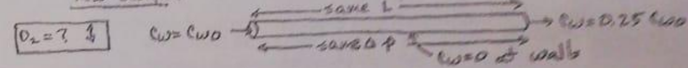
[redacted]

An engineer is injecting a solution containing a toxic waste into narrow tubes of diameter D_1 and length L, from which the toxic waste is removed at the walls. That is, the concentration of the waste c_W is c_{W0} at the entrance of the tubes, and is (0.5 c_{W0}) at the exit of the tubes. (At the tube walls, c_W=0.) The engineer needs to reduce c_W at the exit to 0.25 c_W, and he can change only the diameter of the tubes, not their length. He does know that the fluid is in laminar flow through the tubes and is flowing with a fixed ΔP. What new value of tube diameter D_2 would be required to achieve the desired value of c_W at the tube outlet?

current design:



new design:



In the original experiment laminar flow applied. Since ΔP is fixed, $v \sim R^2$ in laminar flow, $Re = Dv\rho/\mu \sim D D^2 \sim D^3$; thus as D decreas Re decreases and laminar flow still applies.

For laminar flow one next asks whether $(c_{wb2} - c_{wo})/(c_{wm} - c_{wo})$? in the first experiment, this ratio is 0.5, and the new design gives ratio of 0.25; thus the inequality is satisfied.

There fore there are 3 potential ways to solve the problem: Fig 14.3- Fig 14.2-1, and Eq 14.3-17. We can't use Figure 14.3-3, because we don't the actual value of Re (and it may be off the chart). Eq. 14.3-17 gives

$$Sh = Nu_{AB} = 1.86\,(Re\,Sc\,D/L)^{1/3}(\mu_b/\mu_0)^{0.14}$$

← transport" law"; still need ation eq

where we ignore the viscosity term. "Eq. $\cancel{III}$" $Re\,Sc\,\frac{D}{4L}$ gives

$$Sh = Nu_{AB} = \frac{c_{Ab2} - c_{Ab1}}{(c_{A0} - c_{Ab})_{\ln}} \frac{D^2}{4L\mathcal{D}_{AB}} \langle v \rangle = 1.86\,(Re\,Sc\,D/L)^{1/3}$$

For this case, in which $c_{A0} = 0 =$ uniform, $\frac{c_{Ab2} - c_{Ab1}}{(c_{A0} - c_{Ab})_{\ln}} = \ln \frac{c_{A0} - c_{Ab1}}{c_{A0} - c_{Ab2}} = \ln$
$= \ln 2$ for the new design, this term is $\ln 4$.

Combining the two equations gives

$$1.86\,(Re\,Sc\,D/L)^{1/3} = \ln\left(\frac{c_{A0} - c_{Ab1}}{c_{A0} - c_{Ab2}}\right) \frac{D^2}{4L\mathcal{D}_{AB}} \langle v \rangle$$

or

$$\ln\left(\frac{c_{A0} - c_{Ab1}}{c_{A0} - c_{Ab2}}\right) = \frac{1.86\,(Re\,Sc\,D/L)^{1/3}}{D^2 \langle v \rangle / (4L\mathcal{D}_{AB})}$$

Since fluid properties and L are fixed,

$$\ln\left(\frac{c_{A0} - c_{Ab1}}{c_{A0} - c_{Ab2}}\right) \sim (Re\,D)^{1/3} / [D^2 \langle v \rangle] \sim [D^3 D]^{1/3} / [D^2 D^2] \sim (D^4$$

Since the left size is to be changed from $\ln 2$ to $\ln 4$,

$$\frac{\ln 2}{\ln 4} = \frac{(D_1)^{-8/3}}{(D_2)^{-8/3}} = \left(\frac{D_1}{D_2}\right)^{-8/3};\quad \left(\frac{D_1}{D_2}\right) = \left(\frac{\ln 2}{\ln 4}\right)^{-3/8} = (0.5)^{-3/8} = 1.$$

$D_2 = (D_1)\,(0.771)$.