UNIT TWO:
HEAT TRANSFER
(ENERGY TRANSFER)

Fourier's law of conduction (BSL ch. 8)

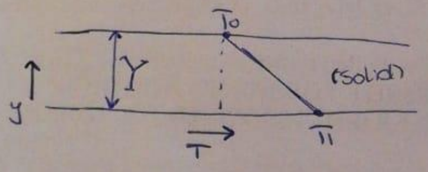
A. 3 modes of heat transfer

conduction
convection
radiation

energy		momentum transfer
conduction	(molecular transport)	viscosity
convection		convection
radiation		[nothing]

te: there is no material that conducts heat as a non-Newtonian fluid transports momentum flux depends nonlinearly on the gradient of a property)

Fourier's law of conduction BSLK 9.2-1



$$q_y \cancel{q_y} = -k \cdot \frac{dT}{dy}$$

k = thermal conductivity

$\alpha \equiv \frac{k}{\rho \hat{C}_p}$ ≡ "thermal diffusivity"

ρ = density
$\hat{C}_p$ = heat capacity / mass

more generally, $\vec{q} = -k \nabla T$ BSLK 9.2-6

gradient

1. Units

q $\frac{J}{m^2 s} = \frac{W}{m^2}$

k $\frac{W}{m \cdot K}$

T K

α m^2/s

y m

(see also BSL pp. 269 ff. and Appendix F) (BSLK p.258 + Appendix E)

References for values of k:

- BSL, p. 269-271 (BSLK pp. 261-263)
- for oils and rock, L. W. Lake, *Enhanced Oil Recovery*, Prentice-Hall, 1989

I. Shell energy balance (BSL ch. 10) BSLK ch.10, BSL1 ch 9

A. General form of balance:

$$\underbrace{\left\{\begin{matrix}\text{transport of thermal}\\ \text{energy in}\end{matrix}\right\} - \left\{\begin{matrix}\text{transport}\\ \text{of thermal}\\ \text{energy out}\end{matrix}\right\}}_{\text{across system boundaries}} + \underbrace{\left\{\begin{matrix}\text{net release}\\ \text{of thermal}\\ \text{energy}\\ \text{(generation)}\end{matrix}\right\} = \left\{\begin{matrix}\text{accumulation}\\ \text{of thermal}\\ \text{energy}\end{matrix}\right\}}_{\text{within system volume}}$$

~~1. Review: macroscopic energy balance~~
~~(cooling of metal ball, I.C.2.d)~~

~~2. Another review: when to use macro balance and~~
~~when shell balance?~~

B. Outline of Shell Energy Balance Approach (BSL ch. 10)

APPROACH

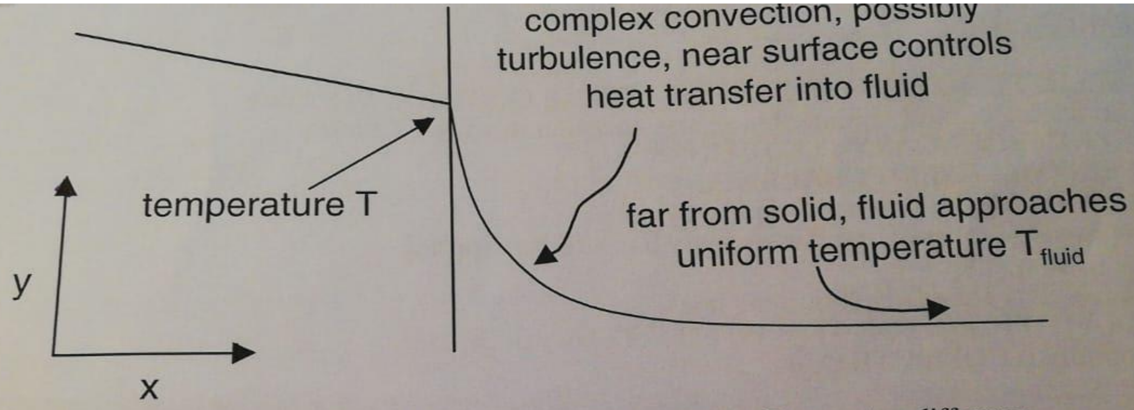
surfaces perpendicular ~~or parallel with~~ to any convection and perpendicular to temperature gradient ∇T

1. SELECT COORDINATE SYSTEM; DEFINE CONTROL VOLUME
   principle: control volume thin in any direction in which T varies
2. STATE BOUNDARY CONDITIONS *
3. PERFORM ENERGY BALANCE **
4. THICKNESS → 0 (→ dif. eq. for q)
   (optional): solve dif. eq. for q, apply b.c. - IF b.c. applies
   to q alone
5. RELATE q TO dT/dx (Fourier's law)
6. SOLVE DIF. EQ. FOR T; APPLY B.C.*
   (optional) COMPUTE Q, etc.

boundary conditions
BSL1: 267

* - BOUNDARY CONDITIONS (see pp. 291-292} in BSL2; BSL 1st ed. p.267
BSLK p.280

1. SPECIFY T AT SURFACE
2. SPECIFY q AT SURFACE
   2a) q = 0 across surface ("perfectly insulated surface")
   2b) q specified in problem statement
   CONTINUOUS ACROSS SOLID/SOLID I.F.



Assume heat flux at solid surface is proportional to Temperature difference between solid surface and fluid far away:

$$q_x|_{\text{bandary}} = h(\overset{\text{of Solid at wall}}{T} - \underset{\text{Far away}}{T_{\text{fluid}}})$$

h = "heat-transfer coefficient" units: $W/(m^2K)$

C. Examples

1. Brief preview of BSL Ch. 9 (1st ed.; corresponding sections in Ch. 10 in 2nd ed. & BSLK)

2. "heat conduction with electrical heat source" (BSL^2 Sect. 10.2) (BSLK 10.6)

final result:

result directly analogous to velocity profile in tube (p. 294)

3. Analogies between heat transfer and momentum transfer

hearing between flat plates	conduction through solid slab
specify (unequal) V at both surfaces	→ specify unequal T on oppasite surfaces
no "source" term (no ΔP, no gravity)	no heat generation
	*flat wire, perfectly insulated on one side, fixed T on other *
lling film	
specify V at one wall, t = 0 at other	specify q on one surface (q = 0); specify T on other
source = gravity →	heat generation (e.g. electrical resistance)
	*annular heating element *
v in annulus	
specify equal (zero) V at both walls	specify equal T on opposite surfaces
ource = $\overset{\Delta P/L}{\Delta P/L}$	electrical or other generation