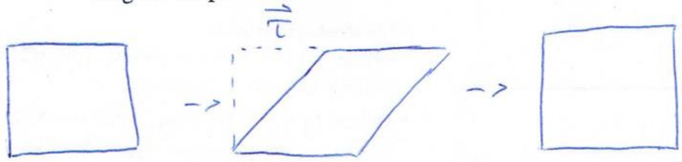
UNIT ONE:
FLUID MECHANICS
(MOMENTUM TRANSFER)

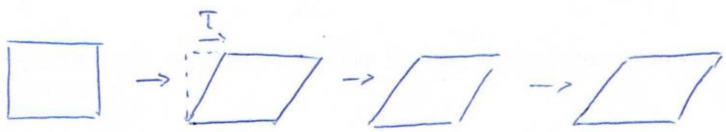
II. Constitutive Equations

A. Difference between fluid and solid

perfectly elastic solid: given shear stress τ --> finite deformation; returns to original shape if force is released



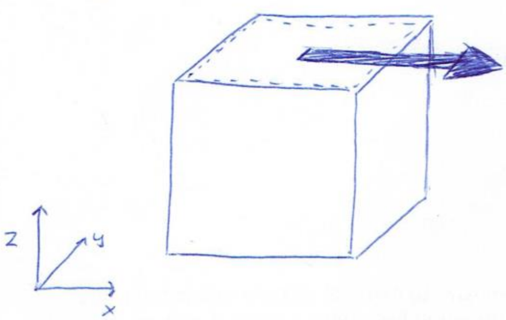
fluid: given shear stress τ --> finite *rate* of deformation; further deformation stops if force is released, but there is no return to original shape (unless an opposite force is applied)



B. Definition of shear stress

Consider body of fluid with small rectangular element of fluid within it

Newton's second law: *force = change of momentum*



shear stress: τ_{yx} force per unit area

- x: force/area in x direction
- y: acting on surface of constant y

τ_{yx}: flux of momentum

- x: flux of x momentum
- y: flux is in y direction

<u>units</u> of shear stress:

represented as flux of momentum: momentum/area/time

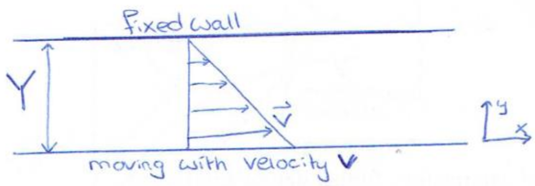
represented as force/area: force/area

- *units are same in either case:* (m / L t^2)

C. Newton's "law" of viscosity

1. Statement of Newton's law

a. thought experiment



• ~~viscosity at wall~~
• velocity at which ~~pate~~ plate moves depends on viscosity
• velocity practically determined by the coaxial cylinder within the annulus

$F/A = -\mu\,(\Delta v_x/\Delta v) = -\mu\,[(0 - V)/(Y - 0)] = \mu\,(V/Y)$

$\tau_{yx} = -\mu(dv_x/dy)$

Newtonian fluids:

b. beware the sign in Newton's law!

c. kinematic viscosity: $\nu = \frac{\mu}{\rho}$

d. units (BSL p. 14; also BSL App. C)

BSLK p.20

[Pa·s or ~~Centipole~~]

centipoise

2. Molecular origins of Newtonian viscosity

BSLK Fig 1.6-1

consider simplest case: ideal gas, small molecules (BSL fig. 1.4.1)

molecules cross streamlines in random motions; they take their momentum with them when they cross

exchange of molecules between streamlines, on average, transfers momentum from fast streamlines to slow ones
(cf. figure (2-2?) from Roberson and Crowe of people moving between conveyor belts)

This momentum transfer is origin of "viscosity" in ideal gases
(situation is more complicated in liquids)

In each exchange,

- distance is short
- amount of momentum transferred is small
  (though there are many exchanges per unit volume per unit time)

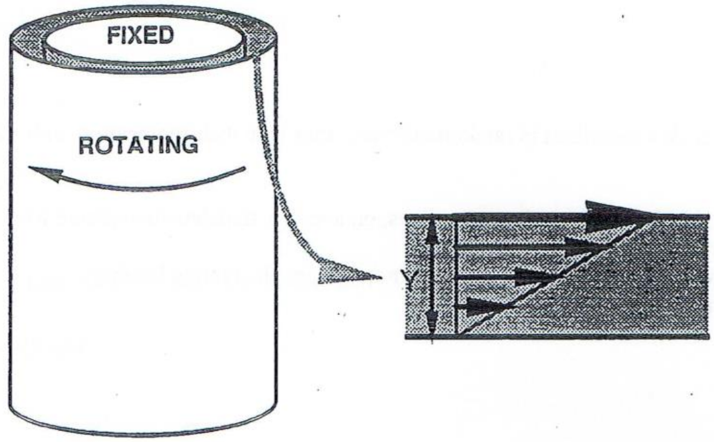
Don't sweat rest of BSL section 1.4

II. C.1.e.

INFINITE PARALLEL PLATES ARE IMPOSSIBLE, OF COURSE.

THEN HOW CAN ONE MEASURE VISCOSITY?

USING CONCENTRIC CYLINDERS. IF GAP WIDTH→0, GAP APPROXIMATES PLANAR GEOMETRY.



IF GAP WIDTH $\not\to$0, NEED CORRECTION FACTORS

. . . THE IDEA BEHIND "FANN" VISCOMETER

II. C. 3 [redacted] 2.5

MAGNITUDES OF μ

CRUDE OILS >> WATER >> GASES

TRENDS OF μ WITH TEMPERATURE AND PRESSURE

PURE LIQUIDS

- μ DECREASES AS TEMPERATURE INCREASES
  - BASIS OF THERMAL E.O.R.
- μ RELATIVELY INDEPENDENT OF PRESSURE

GASES AT LOW PRESSURE

- μ INCREASES AS TEMPERATURE INCREASES
- μ INDEPENDENT OF PRESSURE

GASES IN OR NEAR CRITICAL REGION

- TRENDS OF μ WITH T AND P ARE COMPLEX

CRUDE OILS WITH DISSOLVED GAS

- DISSOLVED GAS REDUCES μ OF OIL
  - PART OF BASIS FOR CO_2 E.O.R.
- μ DECREASES AS P DECREASES, UNTIL BUBBLE POINT REACHED
- μ INCREASES AS P DECREASES FURTHER, AS GAS LEAVES SOLUTION

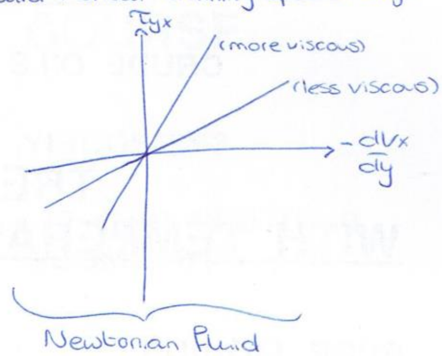
II. D. non-Newtonian fluids (BSL 1st ed.)

1. motivation:

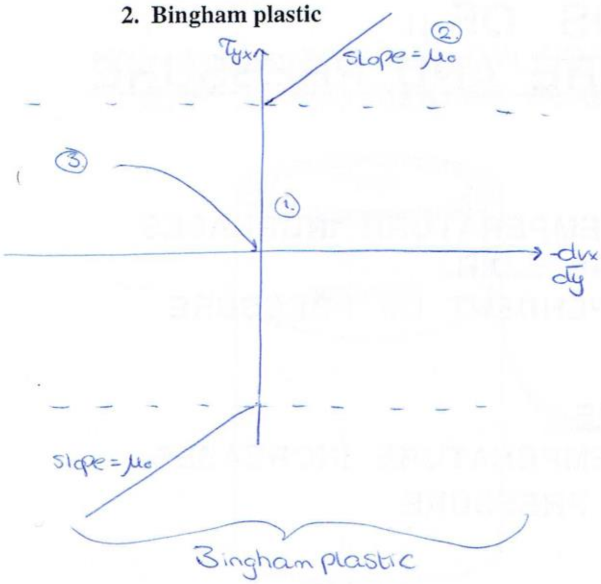
sometimes want fluids that sometimes behave like solids: Bingham Plastic

sometimes want fluids that have low viscosity near well,
high viscosity away from well: Power Law Fluid ('Shear' thinning specifically)

Recall for Newtonian fluid: $\tau_{yx} = -\mu\,(dv_x/dy)$



2. Bingham plastic



① low shear stress relative to yield stress τ_0 → Fluid behaves like elastic solid

② Sufficient shear stress to overcome yield stress and this makes fluid behave like Newtonian fluid.

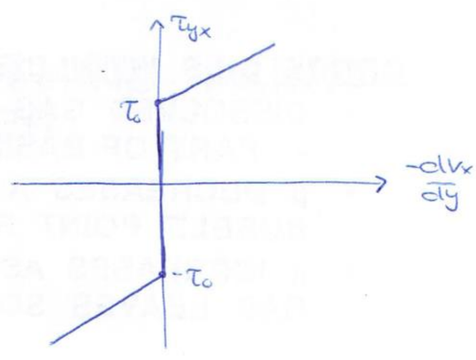
③ If $\tau_{yx} = 0$, then Bingham plastic is a Newtonian fluid

a. equations:

$$\tau_{yx} = -\mu_0 \frac{dv_x}{dy} + \tau_0 \;,\; \text{if } \tau_{yx} \geq \tau_0$$

$$-\frac{dv_x}{dy} = 0 \;,\; \text{for } -\tau_0 \leq \tau_{yx} \leq \tau_0$$

$$\tau_{yx} = -\mu_0 \frac{dv_x}{dy} - \tau_0 \;,\; \text{if } \tau_{yx} \leq -\tau_0$$



(Beware confusion over ± sign in BSL eq. 1.2-2 (1st ed.)

b. two parameters: μ_o, τ_o

μ_o: Plastic Viscosity (not A viscosity). Viscosity is strictly for Newtonian Fluid

τ_o: Yield stress or "Yield Point Yp"

- Temperature of fluid affects the yield stress of a Bingham plastic

units:

μ_o: [Pa·s]

τ_o: [Pa]

Note: if τ_o --> 0, Bingham plastic --> Newtonian fluid

c. examples of fluids with yield stress:

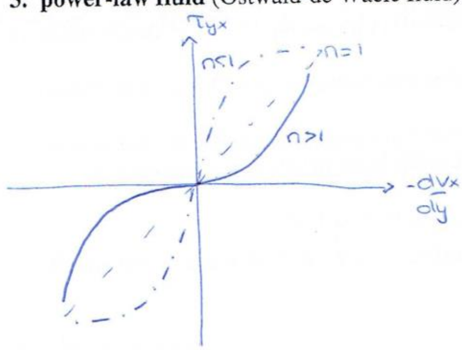
- Peanut butter
- Nutella
- Mayonaise
- Paint
- Wax

d. two asides:

why does honey tear bread, while mayonnaise does not?

why does ketchup get stuck in the neck of a ketchup bottle?

3. **power-law fluid** (Ostwald-de Waele fluid)



Power-Law Fluid
n<1 : "Shear thinning" Fluid
n>1 : "Shear thickening" Fluid
n=1: Newtonian Fluid

a. equation:

$\tau_{yx} = -m \left| \frac{dv_x}{dy} \right|^{n-1} \left(\frac{dv_x}{dy} \right)$, the $\left| \frac{dv_x}{dy} \right|^{n-1}$ term keeps all numbers well-behaved and real numbers.

b. two parameters: m, n

m: Sometimes called 'consistency index'

n: power law index

If n = 1: Newtonian fluid, with $m \equiv \mu$

n < 1: "shear thinning", "pseudoplastic"

n > 1: "shear thickening", "dilatant"

(recall definition of $|x|$:

if $x \geq 0$, $|x| = x$
if $x \leq 0$, $|x| = -x$)

4. effective viscosity of non-Newtonian fluids

a. general definition:

The "effective viscosity" of a non-Newtonian fluid is the viscosity of a Newtonian fluid hypothetical Newtonian fluid that would give the same result

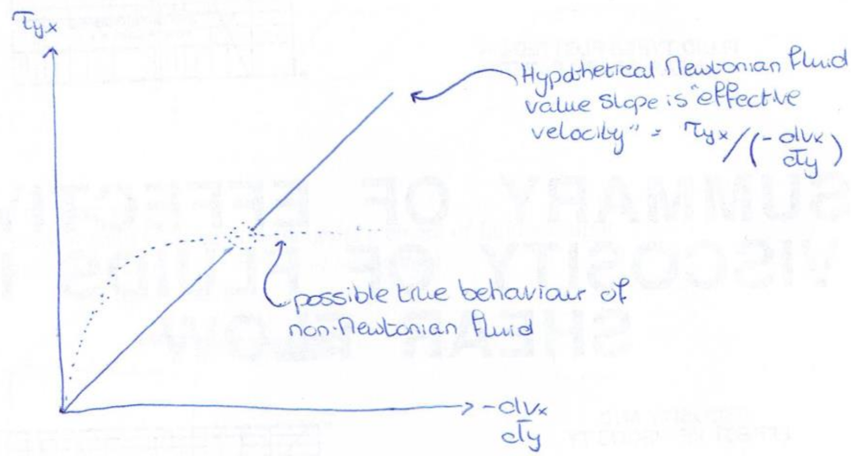
as the real fluid does in the same situation

(Definition applies even if have no idea of true nature of fluid or even of how viscometer works - see homework)

b. effective viscosity for shear flow between parallel plates

The "effective viscosity" of a non-Newtonian fluid in shear flow between parallel plates is the viscosity of a hypothetical Newtonian fluid that would

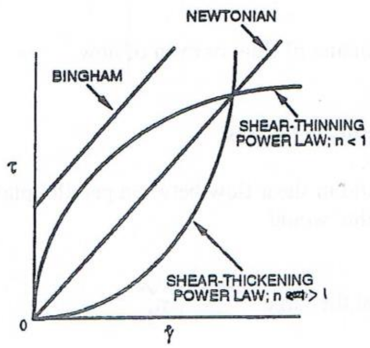
give the same τ_{yx} as the real fluid does at the same $-\frac{dv_x}{dy}$



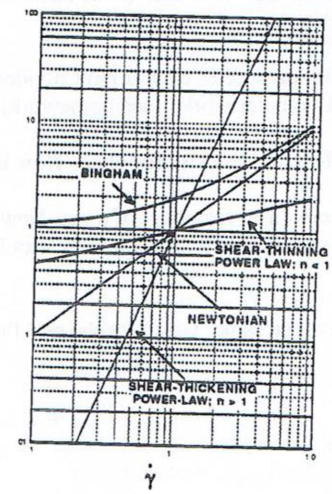
Note; the given value of "effective viscosity" may not apply to other situations:

2.10

SUMMARY OF BEHAVIOR OF DIFFERENT FLUIDS IN SHEAR FLOW



FLUID TYPES PLOTTED ON LINEAR GRAPH PAPER



SUMMARY OF EFFECTIVE VISCOSITY OF FLUIDS IN SHEAR FLOW

VISCOSITY AND EFFECTIVE VISCOSITY