## X". Macroscopic Mechanical Energy Balance

Flow in industrial situations involves not only drag on pipes, but the work of pumps to make fluids flow, turbines to extract work from flow, and the effects of fittings and pipe elbows on drag, all with pressure differences and changes of elevation along the system. There can also be acceleration of the system, but we restrict ourselves to cases of steady state. BSL Fig. 7.0-1 shows a schematic of what may be involved. Note that subscript 1 refers to the entrance to the system and subscript 2 to the exit.

The governing equation in this case is the macroscopic mechanical energy balance, the subject of BSL Sections 7.4 and 7.5. The development begins with Eq. 7.4-1. This can be rewritten as Eq. 7.4-2. The left side of this equation is the accumulation term (unsteady state). If we assume steady state, the equation becomes Eq. 7.4-5: this equation is written in terms of extensive quantities - all the terms have units of energy/time. If one divides by mass flow rate w = Qρ and uses an approximate relation for the compression term, one gets Eq. 7.4-7, which is the equation we will use. Note that the terms here are in units of energy/mass.

If we assume turbulent flow, so that v is approximately uniform across the pipe, the kinetic energy term is simplified, and we get Eq. 7.5-11. The terms in the equation are still in units of energy/mass.

$$\underbrace{\frac{1}{2}(V_2^2 - V_1^2)}_{\text{kinetic energy}} + \underbrace{g(h_2 - h_1)}_{\text{potential energy (gravity)}} + \underbrace{\int_{P_1}^{P_2} \frac{1}{\rho}\, dP}_{\text{pressure-volume energy}} = \underbrace{\underbrace{\hat{W}_m}_{\text{work done on fluid}} - \underbrace{\hat{E}_v}_{\text{dissipation (drag)}}}_{\text{per unit mass}}$$

BSL eq. 7.5-12

Note if the fluid is incompressible, the pressure term is simplified (~~Eq. 7.4-7~~).

$$\frac{P_2 - P_1}{\rho}$$

Note that if the inlet pressure is greater than the outlet pressure, *this term is less than zero*. This is the opposite of the $\Delta\mathscr{P}$ term we used before, which is positive if the inlet pressure is greater than outlet.
Note that here $h_2 > h_1$ if the outlet is higher than the inlet - in effect, the z axis points *up*.

Note also that the work term $\hat{W}_m > 0$ if one puts work in (e.g. a pump)

$\hat{W}_m < 0$ if one gets work out (e.g., turbine)

To get total work in or out, multiply $\hat{W}_m$ by mass flow rate w (which is equal going in or out at steady state)

↳ $= Q\rho$

$\hat{E}_v$ -11 BSLK in Eq. 7.4-7 represents the dissipation in the system. It comprises two terms (Eq. 7.5-10):

dissipation in drag in straight lengths of pipe *i*

f(Re) from chart

$$\sum_i \left(\frac{1}{2}v^2 \frac{L}{R_h} f\right)_i = \sum_i \left(\frac{1}{2}v^2 \frac{4L}{D_h} f\right)_i$$

$R_h \leftarrow$ hydr. radius $\quad$ $\hookrightarrow$ hydr. diam.

$\Sigma$ for all lengths of pipe

Calculation of friction factors in pipes f was the subject of section X of our lecture notes (ch. 6 of BSL)

dissipation in flow through fittings, valves, etc.

$$\sum_i \left(\frac{1}{2}v^2 e_v\right)_i$$

Friction Loss Factor

over all fittings

upstream velocity

$e_v$

The values for friction-loss factors for various cases are given in Table 7.5-1. Note the definition of β in the footnote of this table. Note also that v in this term is the average velocity downstream of the fitting.
Putting all this together, we get

$$\frac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1) + \cancel{\frac{p_2 - p_1}{\rho}} = \frac{P_2 - P_1}{\rho}$$

$$\hat{W}_m - \sum_i \left(\frac{1}{2}v^2 \frac{4L}{D_h} f\right)_i - \sum_i \left(\frac{1}{2}v^2 e_v\right)_i$$

(assume flow is through pipes, fittings, etc. in series & Not always true)

Example 7.5-1 uses the macroscopic mechanical energy balance to solve for the requirement for a pump.

**The Bernoulli Equation**

p.86 BSL2; p. 88 BSLK

(194 BSLK) $\qquad$ (3.3-9 on p. 88 BSLK)

On p. ~~204~~ BSL2 compare Eq. 7.4-7 to the Bernoulli equation, Eq. 3.5-12 on p. 86. Note the important comment on p. ~~86~~: the Bernoulli equation applies where "viscosity [i.e., drag] ~~palys~~ a rather minor role." plays
The mechanical energy balance in Eq. 7.4-10 includes two important additions:

work + drag

what if we don't know Re?
use trial + error with eq. 7.5-11 (8BSLK)
instead of Eq. 6.4-1

## Which equation to use, 6.1-4 (momentum balance in a tube) or 7.5-~~12~~ 11 (macroscopic mechanical energy balance)? (BSLK)

Eq. 6.1-4 is based on a momentum balance on a single tube

involving pressure, gravity, drag on walls
(with the assumption that velocity in = velocity out)

(see derivation of momentum balance, p. __ in lecture notes)

In this equation you have to combine

Eq, 7.5-~~12~~ 11 includes other factors as well:
Velocity in $\neq$ velocity out
(i.e., changes in kinetic energy)
work done on fluid or extracted from it
(in) (out)
multiple lengths of (different) tubing
fittings, valves, elbows, etc

(see Fig. 7.0-1)

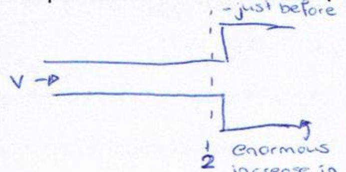
**If those factors are involved in the equation,** you must use the macroscopic mechanical energy balance; **Eq. 6.1-4 simply does not apply**. because it leaves out factors in the given problem.

Eq. 7.5-~~12~~ 11 automatically combines gravity and pressure difference as long as one evaluates $h$ and $p$ correctly at the two surfaces, "1" and "2".

*In fact, because it is more general, one can always use Eq. 7.5-~~12~~ 11, even if those extra factors are not involved. Thus, if it is simpler for you, simply apply Eq. 7.5-~~12~~ 11 to all situations involving flow through pipes or piping systems.*

How to account for fluid velocity exiting a pipe? Is it a kinetic-energy factor or a sudden expansion of diameter (i.e., velocity →0)?

a)

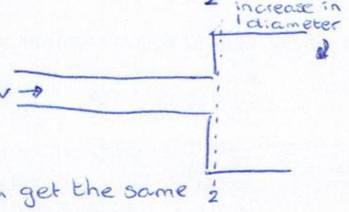



surface 2 just within pipe

no sudden expansion (sudden increase in diameter) at end of system.

$V_2 = V$ in pipe KE includes $V_2^2 = V^2$

b)




surface 2 just past pipe outlet

Because of enormous increase in diameter, $V_2 = 0$

~~eq~~ I do include sudden expansion

~~eq~~ $\tilde{e}_v = \frac{1}{2} v^2$

But, KE ~~term~~ changes because $V_2 = 0$

you get the same answer either way.

Most important:

Be consistent. It's one or the other, but not both.

Does the Macroscopic Mechanical Energy Balance assume turbulent flow? Can we use it if flow is laminar in the pipes?

It does not assume turbulent flow in pipes.

f[Re] chart applies for both laminar flow.

It does assume turbulence in friction loss factors $e_v$: fittings, valves, elbows, etc.

But they would experience turbulence even at fairly low velocities

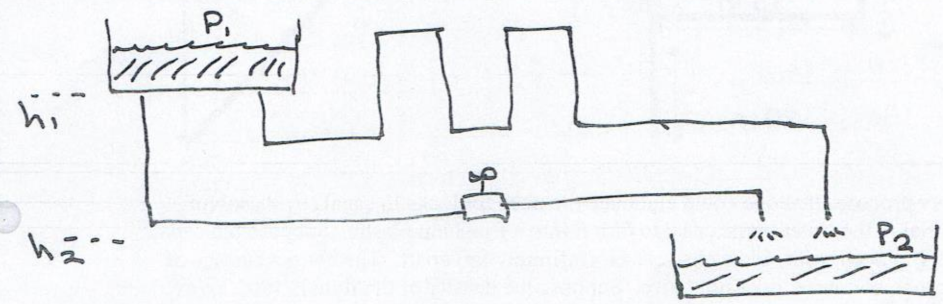
The Macroscopic Mechanical Energy Balance assumes that the pipes, valves, fittings, etc., all apply to a single flow path. The change in height, pressure difference, and work input together must accommodate the **sum** of all the resistances provided by the pipes, valves, fittings, loss of mechanical energy, etc.

$$\frac{1}{2}\left(v_2^2 - v_1^2\right) + g\left(h_2 - h_1\right) + \int_{p_1}^{p_2} \frac{1}{\rho}\, dp = \hat{W}_m - \underbrace{\sum_i \left(2v^2 \frac{L}{D} f(\mathrm{Re})\right)_i}_{\text{sum over all sections of straight conduits}} - \underbrace{\sum_i \left(\frac{1}{2} v^2 e_v\right)_i}_{\text{sum over all fittings, valves, meters, etc.}} \qquad (7.5\text{-}12)$$

This is similar to what we learn in physics about **resistors in series**:

For fluid mechanics, for flow components in series,

What if there were alternate paths the fluid could take to the same destination?




For electrical circuits **in parallel**

Similarly, for the flow problem

We will come back to this analogy when we study heat transfer.

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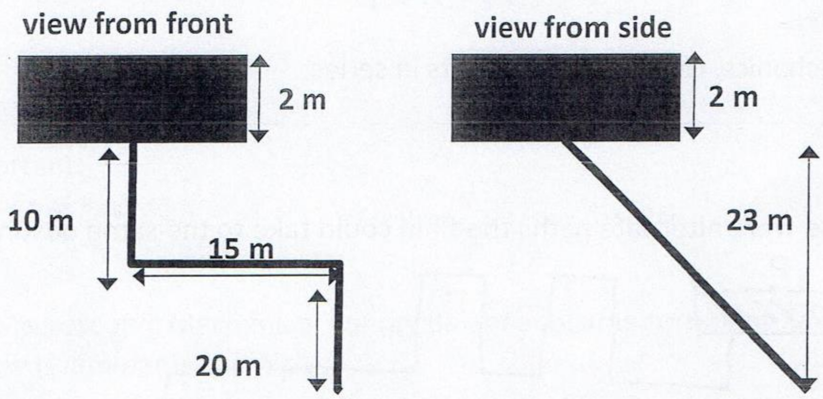
What if we don't know Re?

→ 3. A drainage system is designed to carry rainwater off the top of a hill to a river below. The pipe is 0.2 m in diameter, and its "height of protuberances" 0.0008 m. The properties of water are given in problem 2. There are two sharp (not rounded) 90° elbows and in the pipe plus a sharp (not rounded) constriction at the entrance, as shown below. The pipe discharges into the air above the river. The pipe comprises 10, 15 and 20-m segments (going down the slope, sideways, and down the slope), but because it is set on the side of the hill the change in elevation along the pipe is only 23 m. At the top of the pipe is a reservoir of water of depth 2 m. An engineer wants to know the water velocity through the pipe.

a. Write out the equation that must be satisfied by the velocity in the pipe. Plug in all the numbers you can into this equation.
b. Solve this equation for velocity in the pipe. Start by assuming that the flow is highly turbulent (very large Re).

(35 points)




4. Rocky proposes that one could eliminate the need for locks in canals by dissolving material in the water in the canal to turn it into a Bingham plastic. Suppose the canal were 1 m deep, with side walls very far (infinitely far) apart. The bottom surface of the canal, of course, does not move. Suppose the density of the fluid is 1000 kg/m$^3$. Suppose the canal tilts at an angle 89.8° to the vertical (i.e., it is close to horizontal). What yield stress $\tau_o$ would be required to prevent flow of the Bingham plastic downward through the canal?

(10 points)

<u>properties of water</u>

$\mu = 0.001$ Pa s $\quad\quad \rho = 1000$ kg/m$^3$

(level 1)

3. One can do this problem two ways: a) take the inlet at the bottom of the tank, just above the abrupt construction. The $p_1$ must account for the weight of 2 m of water above. b) Take level 1 at top of tank.* That is what I do here.

7.5-11 (BSLK)
Eq. 7.5-10 applies. Let's go term by term:

$v_1 \approx 0 \quad v_2 = V^2$ (V = velocity in pipe) $\qquad \frac{1}{2}V^2$

$z_2 - z_1 = -(23+2) = -25$ m. $\qquad -25(9.8)$

$\frac{p_2 - p_1}{\rho} = \frac{1\,atm - 1\,atm}{\rho} = 0 \qquad 0$

$\hat{W}_u = 0$ No work in or out $\qquad 0$

pipes $L = 10 + 15 + 20 = 45$ ; $R_h = D/4 = 0.05 \qquad -\frac{1}{2}V^2 \frac{45}{0.05} f$

fittings (see table 7.5-1)

sudden contraction @ bottom of tank $\qquad -\frac{1}{2}V^2(0.45)$

2 sharp elbows 90°: take middle value $\qquad -\frac{1}{2}V^2(1.6) \times 2$

(any value between 1.3 and 1.9 is OK.)

$$\frac{1}{2}V^2 - 25(9.8) + 0 = 0 - \frac{1}{2}V^2 \frac{\overset{45}{\cancel{50}}}{0.05} f - \frac{1}{2}V^2\,(3.65) \quad \text{rearranging}$$

$$245 = V^2\left[\frac{1}{2} + \frac{1}{2}\frac{45}{0.05}f + \frac{3.65}{2}\right] = \left[2.325 + 450f\right]V^2 \qquad \boxed{\text{Eq. I}}$$ (see over)

b) Since we don't know V, we don't know Re or f. But as suggested we start by assuming Re very large. $\frac{k}{D} = \frac{0.0008}{0.2} = 0.004$. As long as $Re \gtrsim 3 \cdot 10^5$, $f \approx 0.007$. (Fig 6.2.2, 2nd ed. BSL.

$$245 = \left[2.325 + 450(0.007)\right]V^2 = \left[2.325 + 3.15\right]V^2 = 5.525\,V^2$$

$$6.7 = V \qquad \left[6.7\ m/s\right]$$

Need to check Re + f, + maybe repeat.

$Re = \frac{(0.2)\,6.7\,(1000)}{0.001} = 1.3 \cdot 10^6$. Assumption was good.

We're done.

[The pipe could carry $\pi(0.1)^2\, 6.7 = 0.21\ m^3/s$ water.]

SEE MORE ON NEXT PAGE

*If one takes level 1 as bottom of tank, $z_2 - z_1 = -23$ m, and $\frac{p_2 - p_1}{\rho} = 2(9.8)$, same final result.

4. This is a falling film again. This time the derivation in Sect 2.2 applies in the Eq. (BSL 1st ed.) [Eq. 2.2-13 in 2nd ed.]. [2.2-15 in BSL

$$\tau_{xz} = \rho g \cos\beta\, x = 1000(9.8)(0.00349)\, x$$
$$= 34.2\, x$$

Flow occurs if $\tau_{xz} > \tau_0$ anywhere. $\tau_{xz}$ has its largest value at $x = 1$ m (the bottom). Rocky needs
$$\tau_0 > 34.2 \text{ Pa}$$

More on problem 3:

More generally, because we don't know V or Re, we would solve for V by iteration. Eq. I on previous page could be rearranged:
$$V = \left(\frac{245}{21325 + 4150\,f}\right)^{1/2}$$
This eq. plays the role of Eq. 6.1-4 in the trial-and-error process.* Eq. 6.1-4 is based on a momentum balance with no kinetic energy effects, fittings, or work done on or by the system. It therefore does not apply to this problem.

* that is: guess V, calc Re, find f from chart, calculate V from Eq. 1, repeat.