Midterm Test Linear Algebra CSE1205
14 March 2019, 09.00 – 11.00 uur

- No calculators are allowed.
- **Credits**: 1 point per question.
- The **final score**: sum and multiply by $\frac{2}{3}$.

---

1. Let $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, and $\mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$. Find the value of h such that $\mathbf{w} = \begin{bmatrix} 4 \\ 3 \\ h \\ 0 \end{bmatrix}$ lies in Span $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$.

A. -4 **B.** -3 **C.** -2 **D.** -1 **E.** 0 **F.** 1 **G.** 2 **H.** 3

2. Suppose A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions. Given a vector $\mathbf{b} \in \mathbb{R}^n$, the equation $A\mathbf{x} = \mathbf{b}$ has (choose the most correct answer)

A. 0 solutions
B. 1 solution
C. ∞ many solutions
D. 0 or 1 solution
E. either 0 or ∞ many solutions
F. either 1 or ∞ many solutions
G. 0, 1 or ∞ many solutions
H. none of the above

3. Calculate the inverse A^{-1} of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ if it exists, and determine the sum of the entries of A^{-1} (choose U if the inverse of A does not exist):

A. U **B.** -3 **C.** -2 **D.** -1 **E.** 0 **F.** 1 **G.** 2 **H.** 3

4. Let $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 - x_3 \\ x_1 - x_2 + x_3 \end{bmatrix}$ and $S\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right) = \begin{bmatrix} y_1 - 2y_2 \\ y_1 + y_2 \\ y_1 - y_2 \end{bmatrix}$.

The standard matrix of $S \circ T$ is given by

A. $\begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & -1 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 & -4 \\ 1 & 1 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & 0 \\ 1 & -4 \end{bmatrix}$ **D.** $\begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$ **F.** $\begin{bmatrix} -1 & 3 & -3 \\ 2 & 0 & 0 \\ 0 & 2 & -2 \end{bmatrix}$ **G.** $\begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & 2 \\ -3 & 0 & -2 \end{bmatrix}$ **H.** $\begin{bmatrix} -1 & 2 & 0 \\ -3 & 0 & -2 \\ 3 & 0 & 2 \end{bmatrix}$

5. Let A be a 4×7 matrix. What are all the possible values for $\dim \text{Nul}(A)$?

A. 0, 1, 2, 3
B. 1, 2, 3, 4
C. 3, 4, 5, 6
D. 4, 5, 6, 7
E. 0, 1, 2, 3, 4
F. 2, 3, 4, 5, 6
G. 3, 4, 5, 6, 7
H. 0, 1, 2, 3, 4, 5, 6, 7

6. Calculate the reduced echelon form U of the matrix $A = \begin{bmatrix} -1 & 0 & 1 & -3 & -2 \\ 0 & 2 & 0 & 0 & 1 \\ -2 & 2 & 3 & 1 & -3 \end{bmatrix}$.

What are the last two columns of U?

A. $\begin{bmatrix} -3 & -2 \\ 0 & 1 \\ 7 & 0 \end{bmatrix}$ **B.** $\begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 7 & 0 \end{bmatrix}$ **C.** $\begin{bmatrix} 3 & 2 \\ 0 & \frac{1}{2} \\ 7 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 10 & 2 \\ 0 & \frac{1}{2} \\ 7 & 0 \end{bmatrix}$ **E.** $\begin{bmatrix} 10 & 2 \\ 0 & 1 \\ 7 & 0 \end{bmatrix}$ **F.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

7. Find the determinant of the matrix $\begin{bmatrix} 1 & 0 & 3 & -1 \\ 1 & 0 & 2 & 0 \\ 2 & -2 & 1 & 4 \\ 2 & 0 & 1 & 0 \end{bmatrix}$.

A. -6 **B.** -4 **C.** -2 **D.** 0 **E.** 2 **F.** 4 **G.** 6 **H.** 7

8. Consider the basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ of $\mathbb{R}^3$, with $\mathbf{b}_1 = \mathbf{e}_1, \mathbf{b}_2 = \mathbf{e}_1 + \mathbf{e}_2, \mathbf{b}_3 = \mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3$ and the vector $\mathbf{x} = 4\mathbf{e}_1 + 5\mathbf{e}_2 + 6\mathbf{e}_3$. What is the value of c_1 in $[x]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$?

A. -3 **B.** -2 **C.** -1 **D.** 0 **E.** 1 **F.** 2 **G.** 3 **H.** 4

9. Let A, B, C be $n \times n$ *invertible* matrices. Which of the following statements is not always true?

A. $AB = CA \implies B = C$ **B.** $(A^T)^{-1} = (A^{-1})^T$
C. $A(B + C) = AB + AC$ **D.** $AB = AC \implies B = C$
E. $(AB)^{-1} = B^{-1}A^{-1}$ **F.** $A(BC) = (AB)C$

10. Let A, B, C be 4×4 matrices satisfying $AB = -B^2C$. Assume that $\det B = 3$ and $\det C = 2$. Calculate the determinant of A.

A. -18 **B.** -6 **C.** -3 **D.** 3 **E.** 6 **F.** 18

11. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ a linear transformation satisfying

$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Calculate $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$:

A. $\begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$ **B.** $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **D.** $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ **E.** $\begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix}$ **F.** $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ **G.** $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ **H.** $\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$

12. Which of the statements is true for all square matrices A?

(I) $\det(AA^TA) = (\det A)^3$

(II) $\det(kA) = k \det A$ for any scalar k

A. both are true **B.** only (I) is true **C.** only (II) is true **D.** both are false

13. Which of the statements is always true?

(I) $\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ is linearly dependent $\Longrightarrow$ $\mathbf{b}_p$ is a linear combination of $\mathbf{b}_1, \ldots, \mathbf{b}_{p-1}$.

(II) Any set of $n+1$ vectors in $\mathbb{R}^n$ is linearly dependent

A. both are true **B.** only (I) is true **C.** only (II) is true **D.** both are false

14. Find the standard matrix of the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects vectors in the x_1-axis and then rotates counter clockwise over $\frac{\pi}{6}$ around the origin.

A. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **B.** $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **C.** $\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

D. $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$ **E.** $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **F.** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

15. The matrix $A = [\mathbf{a}_1\, \mathbf{a}_2\, \mathbf{a}_3\, \mathbf{a}_4\, \mathbf{a}_5] = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 2 & 4 & 1 & 3 & 0 \\ 3 & 6 & 1 & 4 & 1 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix}$ is row equivalent to $U = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Consider the sets $\mathcal{B}_1 = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$, $\mathcal{B}_2 = \{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_5\}$, and $\mathcal{B}_3 = \{\mathbf{a}_1, \mathbf{a}_4, \mathbf{a}_5\}$. Which of these sets of vectors is/are a basis for Col A?

A. $\mathcal{B}_1, \mathcal{B}_2$ and $\mathcal{B}_3$ **B.** only $\mathcal{B}_1$ and $\mathcal{B}_2$
C. only $\mathcal{B}_1$ and $\mathcal{B}_3$ **D.** only $\mathcal{B}_2$ and $\mathcal{B}_3$
E. only $\mathcal{B}_1$ **F.** only $\mathcal{B}_2$
G. only $\mathcal{B}_3$ **H.** none of these